MPM2D Review Day 1 Name: Solution.

Evaluating Expressions and Solving Equations

1. Write each expression as a single power, then evaluate:

a) $5^3 \times 5^5 = 5^8 = 390{,}625$

b) $\dfrac{6^9}{6^3 \times 6^2} = \dfrac{6^9}{6^5} = 6^4 = 1296$

c) $3^7 \times 9 \div 3^4 = \dfrac{3^7 \times 3^2}{3^4} = \dfrac{3^9}{3^4} = 3^5$

$= 243$

d) $(3^2)^3 \times (3^3)^0 = 3^6 \times 3^0$

$= 3^6 \times 1 = 3^6$

$= 729$

2. Expand and simplify each of the following:

a) $(16x^2 - 3x + 11) + (2 + x^2)$

$= 16x^2 - 3x + 11 + 2 + x^2$

$= 17x^2 - 3x + 13$

b) $(4m - 3m^2) - (7m^2 - m)$

$= 4m - 3m^2 - 7m^2 + m$

$= -10m^2 + 5m$

c) $3(x^2 + x) - 4(x^2 + 2x)$

$= 3x^2 + 3x - 4x^2 - 8x$

$= -x^2 - 5x$

3. Solve for the unknown variable:

a) $5x - 4 = 21$

$5x = 21 + 4$

$5x = 25$

$x = 5$

b) $12 - 3y = 6 + 2y$

$-3y - 2y = 6 - 12$

$-5y = -6$

$y = \dfrac{6}{5}$

c) $\dfrac{x}{2} + \dfrac{1}{5} = 4x - \dfrac{1}{3}$ → multiply through by 30

$30\left(\frac{x}{2}\right) + 30\left(\frac{1}{5}\right) = 30(4x) - 30\left(\frac{1}{3}\right)$

$15x + 6 = 120x - 10$

$15x - 120x = -10 - 6 \Rightarrow -105x = -16 \therefore x = \dfrac{16}{105}$

d) $\dfrac{1}{4}x + 3 = 2(2x + 1)$ → multiply by 4

$4\left(\frac{1}{4}x\right) + 4(3) = 8(2x + 1)$

$x + 12 = 16x + 8$

$x - 16x = 8 - 12$

$-15x = -4 \Rightarrow x = \dfrac{4}{15}$

4. Evaluate each expression where $a = -2$.

a) $8 - 5a$

$= 8 - 5(-2)$

$= 8 + 10$

$= 18$

b) $-a^2 + a + 2$

$= -(-2)^2 + (-2) + 2$

$= -4 - 2 + 2$

$= -4$

c) $\dfrac{3a-2}{a}$

$= \dfrac{3(-2) - 2}{-2}$

$= \dfrac{-6-2}{-2}$

$= \dfrac{-8}{-2} = 4$

5. Evaluate each expression where $c = -4$ *and* $d = -3$.

a) $-2cd$

$= -2(-4)(-3)$

$= -24$

b) $2c - 2d$

$= 2(-4) - 2(-3)$

$= -8 + 6$

$= -2$

c) $(c + 3d)^2$

$= (-4 + 3(-3))^2$

$= (-4 - 9)^2$

$= (-13)^2 = 169$

d) $2d^2 - 3c + 5$

$= 2(-3)^2 - 3(-4) + 5$

$= 2(9) + 12 + 5$

$= 18 + 12 + 5$

$= 35$

e) $(c - 2)(8 + d)$

$= (-4 - 2)(8 - 3)$

$= (-6)(5)$

$= -30$

6. Solve the following equations **algebraically.** Be sure to show all your work. Answers should be expressed as fractions in lowest terms, if necessary.

a) $6p + 5 = 23$

$6p = 23 - 5$

$6p = 18$ divide by 6.

$p = 3$

b) $5x + 14 - 3x = 4x + 20$

$2x + 14 = 4x + 20$

$2x - 4x = 20 - 14$

$-2x = +6$

$x = -3$

c) $5(2x - 1) - 7 = 3(1 - 2x) + 17$

$10x - 5 - 7 = 3 - 6x + 17$

$10x - 12 = -6x + 20$

$10x + 6x = 20 + 12$

$16x = 32$

$x = 2$

d) $\frac{x-3}{2} = \frac{x+1}{4} + 3$ multiply by 4

$4\left(\frac{x-3}{2}\right) = 4\left(\frac{x+1}{4}\right) + 3(4)$

$2(x - 3) = x + 1 + 12$

$2x - 6 = x + 13$

$2x - x = 13 + 6$

$x = 19$

e) Verify your solution to question 6b. Show your work below.

LS	RS
$\frac{x-3}{2}$	$\frac{x+1}{4} + 3$
$\frac{19-3}{2}$	$\frac{19+1}{4} + 3$
$\frac{16}{2}$	$\frac{20}{4} + 3$
8	$5 + 3$
	8

LS = RS

$\therefore x = 19$

Day 2

MPM2D

Review: Writing Equations and Graphing Lines

$$slope = \frac{rise}{run}; \quad slope = \frac{y_2 - y_1}{x_2 - x_1}; \quad slope = \frac{\Delta y}{\Delta x}; \quad y - intercept = (0, b); \quad y = mx + b$$

1) Determine the equation of the line with a slope of $\frac{3}{2}$ and passes through the point $(-3,7)$.

$y = \frac{3}{2}x + b$ sub $x = -3$ $y = 7$

$7 = \frac{3}{2}(-3) + b$

$7 = -\frac{9}{2} + b \Rightarrow 7 + \frac{9}{2} = b$

$\frac{14}{2} + \frac{9}{2} = b \Rightarrow b = \frac{23}{2}$

$\therefore y = \frac{3}{2}x + \frac{23}{2}$

2) Determine the equation of the line that passes through the points

a) $(4,-1)$ and $(7,8)$. $m = \frac{8+1}{7-4} = \frac{9}{3} = 3$

$y = 3x + b$ sub $x = 7, y = 8$

$8 = 3(7) + b$

$8 = 21 + b$

$b = -13$ $\therefore y = 3x - 13$

b) $(-3,-5)$ and $(2,-3)$ $m = \frac{-3+5}{2+3} = \frac{2}{5}$

$y = \frac{2}{5}x + b$

$-3 = \frac{2}{5}(2) + b$

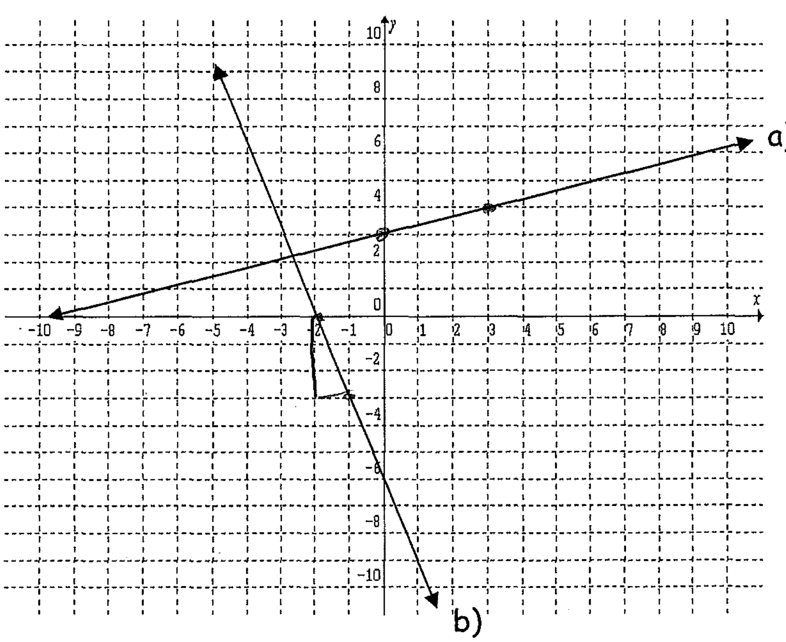
$-3 = \frac{4}{5} + b \Rightarrow -3 - \frac{4}{5} = b$

$\frac{-15}{5} - \frac{4}{5} = b$

$b = \frac{-19}{5}$

$\therefore y = \frac{2}{5}x - \frac{19}{5}$

3) Determine the equation of the following lines.



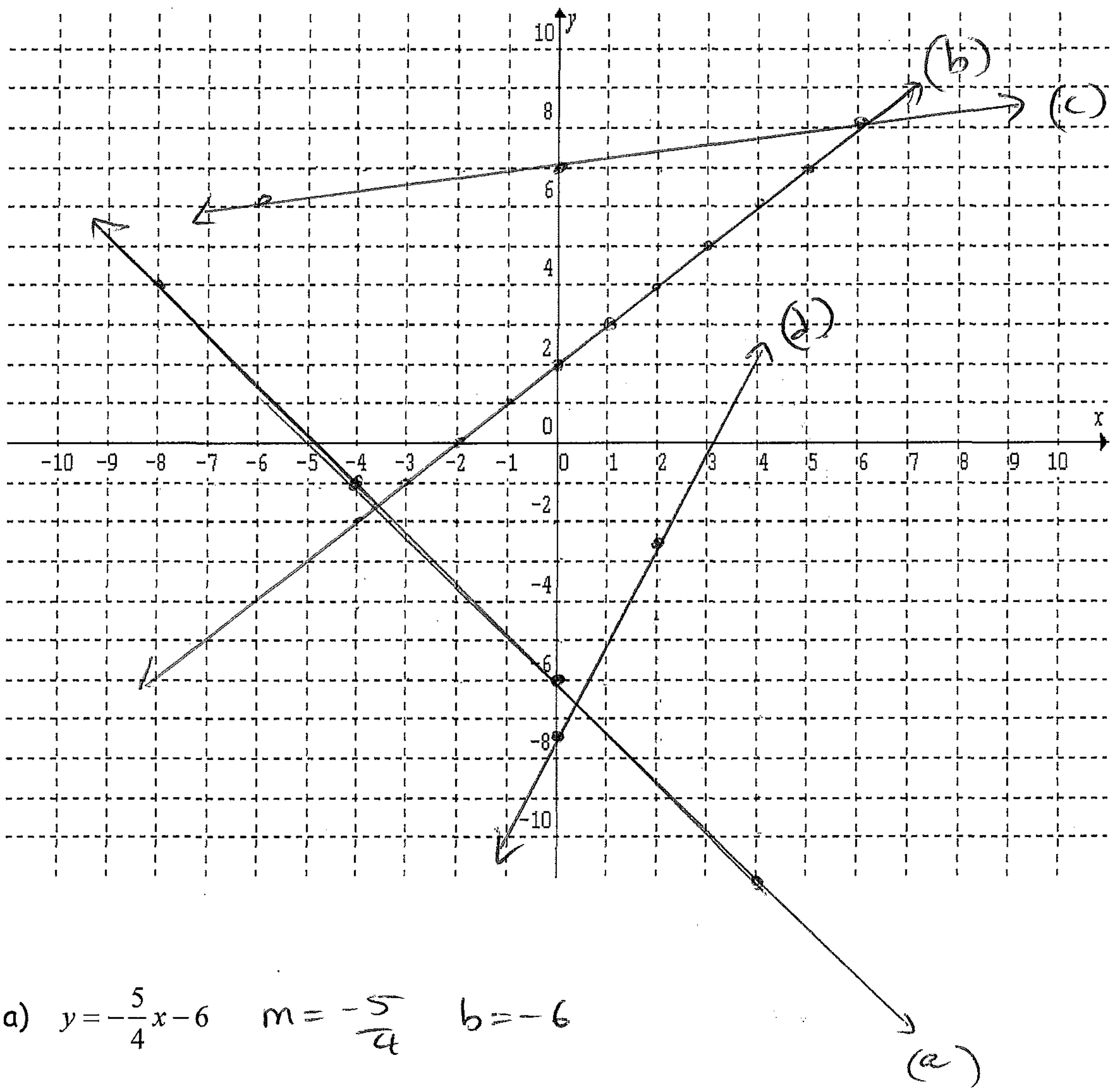
a) $m = \frac{1}{3}$ $b = 3$

$\therefore y = \frac{1}{3}x + 3$

b) $m = \frac{-3}{1}$ $b = -6$

$\therefore y = -3x - 6$

4) Graph each of the following lines



a) $y=-\frac{5}{4}x-6$ $\quad m=\frac{-5}{4}$ $\quad b=-6$

b) $y=x+2$ $\quad m=\frac{1}{1}$ $\quad b=2$

c) $y=\frac{1}{6}x+7$ $\quad m=\frac{1}{6}$ $\quad b=7$

d) $5x-2y=15$

$-2y=-5x+15$

$y=\frac{5}{2}x-\frac{15}{2}$

$=\frac{5}{2}x-7.5$

5) Write the equation for each relation

a) Slope of -3 and y-intercept of 10

$y = -3x + 10$

b) $m = -\frac{7}{16}$ and y-intercept is -1

$y = -\frac{7}{16}x - 1$

c) slope of 4 and passing through (0, 7)

y-int

$y = 4x + 7$

e) horizontal and passing through the point (-5, 18)

$y = 18$

d) $m = \frac{9}{2}$ and passing through (-1, 4)

$y = \frac{9}{2}x + b$

$4 = \frac{9}{2}(-1) + b$

$4 = -\frac{9}{2} + b$

$b = 4 + 9/2 = \frac{17}{2} \quad \therefore y = \frac{9}{2}x + \frac{17}{2}$

f) f) passing through the points (2, 5) and (0, 9)

$m = \frac{9-5}{0-2} = \frac{+4}{-2} = -2$

$y = -2x + 9$

y-int

g) passing through the points (-3, 7) and (2, 4)

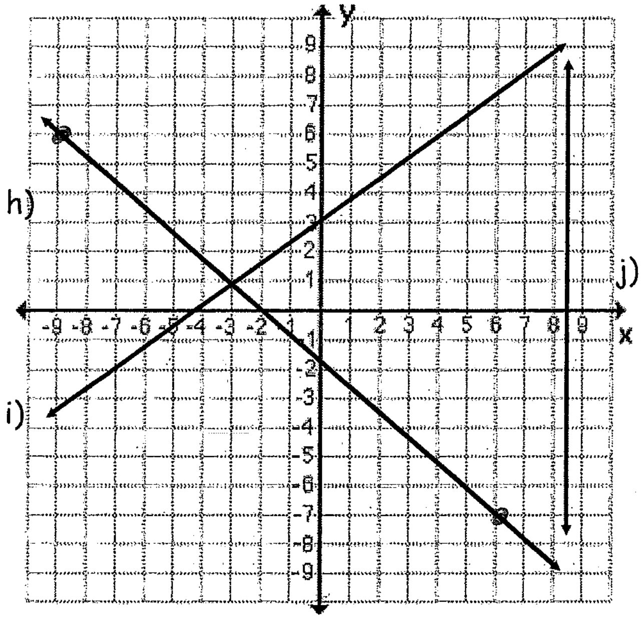
$m = \frac{4-7}{2-(-3)} = \frac{-3}{5}$

$y = -\frac{3}{5}x + b$

$4 = \frac{-3}{5}(2) + b$

$4 = -\frac{6}{5} + b$

$b = 4 + \frac{6}{5} = \frac{20}{5} + \frac{6}{5} = \frac{26}{5} \quad \therefore y = \frac{-3}{5}x + \frac{26}{5}$



h) $y = \frac{-13}{15}x - 1.8$

i) $y = \frac{2}{3}x + 3$

j) $x = 8.5$

h) $(6,-7)$ $(-9,6)$

$m = \frac{6+7}{-9-6} = \frac{13}{-15}$

6) Graph each of the following linear relations. (Feel free to use some scrap paper)

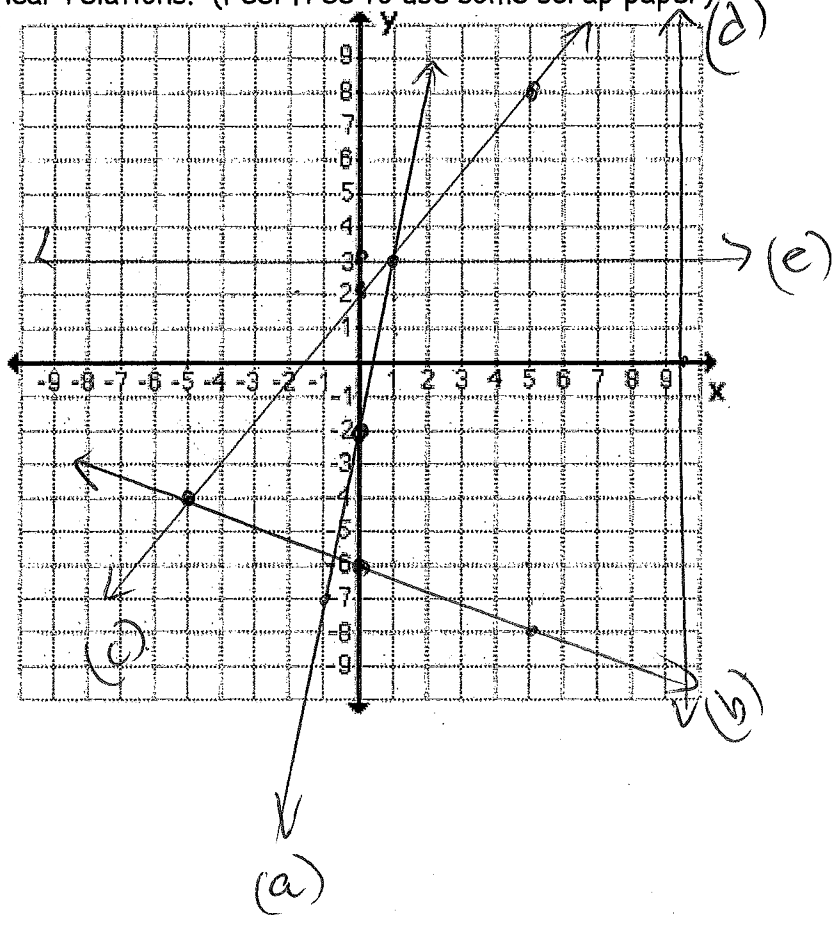
a) $y = 5x - 2$

b) $y = -\frac{2}{5}x - 6$

c) $y = \frac{6}{5}x + 2$

d) $x = \frac{19}{2}$

e) y=3



10 Academic
Day 3: Solving a System GRAPHICALLY

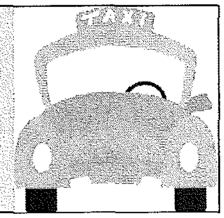
Date:
Unit 1: Linear Systems

Linear Systems - Graphing

WARM UP: Intersecting Lines

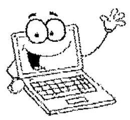
Go-Go Taxi charges $5 to ride their taxi plus $0.30/km.
Take-Me-There Taxi charges $8 to ride, plus $0.20/km.

Express each scenario as a linear equation, where x represents the number of kilometres and y represents the total charge.



Go-Go Taxi: $y =$ 0.30 $x +$ 5

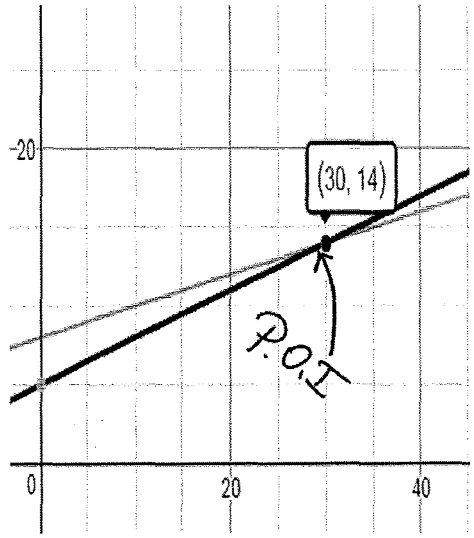
Take-Me-There Taxi: $y =$ 0.20 $x +$ 8



Download **DESMOS** app or go to www.desmos.com

1. Using the graphing calculator, sketch the two graphs on the grid provided.

2. Touch/click on the point of intersection (P.O.I) and determine the coordinates. Label this point on your graph.



A linear system:

① $y = 0.30x + 5$
② $y = 0.20x + 8$

KEY CONCEPTS

- When 2 or more equations are used to model a problem, it is called a system of linear equations. A system of linear equations is simply 2 or more lines intersecting never (||), once, or always (same line). A linear system with two unknowns consists of 2 (or more) linear equations involving 2 variables.
- A solution to a linear system is an ordered pair, (x, y), that satisfies (LS=RS) all the equations in the system.
- If there is a single solution to the linear system, it is represented by the point of intersection of the 2 lines.
- There are several methods to solve linear systems: guess and check, graphing, substitution, and elimination.

Method 1: Guess and Check

To determine whether a point (x, y) is a solution to a linear system using this method, the x and y values must be substituted into the left and right sides of both equations. If same for both equations, then (x, y) is a solution.

Ex1. Determine whether (30, 14) is a solution to the linear system above.

① $y = 0.30x + 5$

LS	RS
y	$0.30x+5$
14	$=0.3(30)+5$
	$=9+5$
	$=14$

✓ LS = RS

② $y = 0.20x + 8$

LS	RS
y	$0.20x+8$
14	$=0.20(30)+8$
	$=6+8$
	$=14$

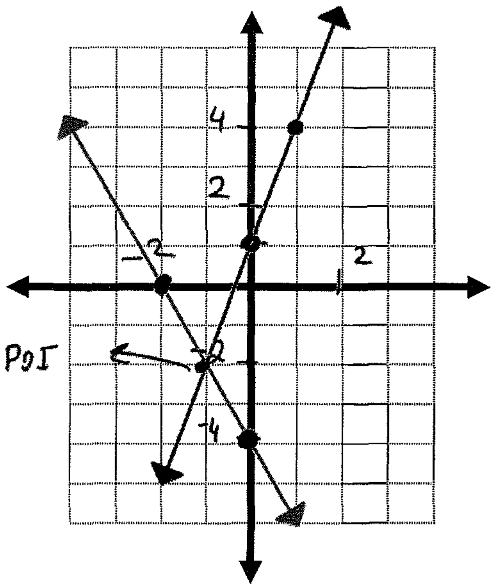
✓ LS = RS

∴ (30, 14) is the solution

Method 2: Graphing

To determine the solution to a linear system using this method, both lines are graphed and the solution is the point of intersection (x, y) of the two lines. Solutions found using this method must be checked by substituting the x and y values into the left and right sides of both original equations.

Ex2. Find the solution to the linear system below ① y = 3x + 1
② 6x + 3y = -12



slope y-int

intercepts

① $y=3x+1$	② $6x+3y=-12$
slope $=3 \Rightarrow \frac{rise}{run}=\frac{3}{1}$	x-int
y-int $=1$	$6x+3(0)=-12$
	$\frac{6x}{6}=\frac{-12}{6}$
	$\boxed{x=-2}$
	y-int
	$6(0)+3y=-12$
	$\frac{3y}{3}=\frac{-12}{3}$
	$\boxed{y=-4}$

The P.O.I is (-1, -2)

Check solution in left and right sides of both equations:

sub
$x=-1$
$y=-2$

Equation ① y = 3x + 1

LS	RS
y	$3x+1$
-2	$=3(-1)+1$
	$=-3+1$
	$=-2$

LS = RS ✓

Equation ② 6x + 3y = -12

LS	RS
$6x+3y$	-12
$=6(-1)+3(-2)$	
$=-6-6$	
$=-12$	

LS = RS

∴ The sol. is (-1, -2)

10 Academic
Day 2: Graphing Linear Relationships

Date:

A linear relationship can be written in the standard form **Ax + By + C = 0** and slope y-intercept form **y = mx + b**

Graph: 2x - y - 1 = 0

METHOD 1: SLOPE and Y-INTERCEPT

Step1: Rearrange the equation in slope y-intercept form as y = mx + b

$2x - y - 1 = 0$
$+y \quad\quad +y$

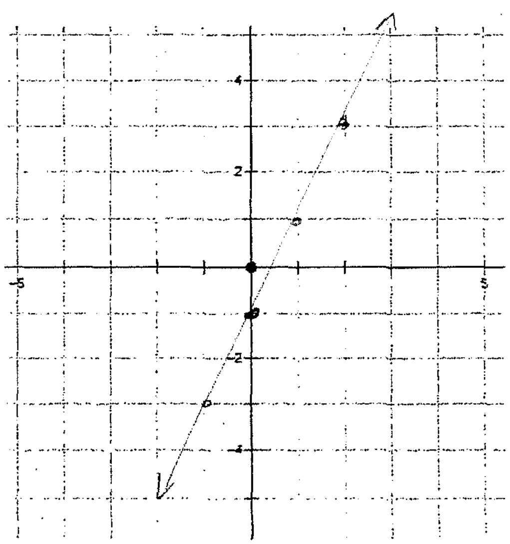
$y = 2x - 1$

Step2: Determine the slope (m) and y-intercept (b)

Slope (m) = 2 and y-intercept (b) = -1

$2 = \frac{2}{1}$ 1 Right 2 up. oR 1 Left 2 down

Step3: Plot the y-intercept first. From there, move right (always) as much as run, then move up if slope + or down if slope - to find a second point and connect with an extended line.



METHOD 2: USING X AND Y - INTERCEPTS

Step1: To find the x-intercept, let y = 0 and solve for x.

$2x - y - 1 = 0$ (sub $y = 0$)

$2x - 0 - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$ $\therefore$ x-int is $\frac{1}{2}$

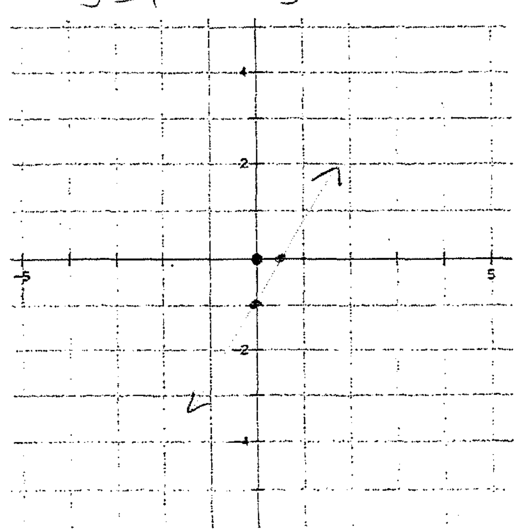
Step2: To find the y-intercept, let x = 0 and solve for y.

$2x - y - 1 = 0$ (sub $x = 0$)

$2(0) - y - 1 = 0$

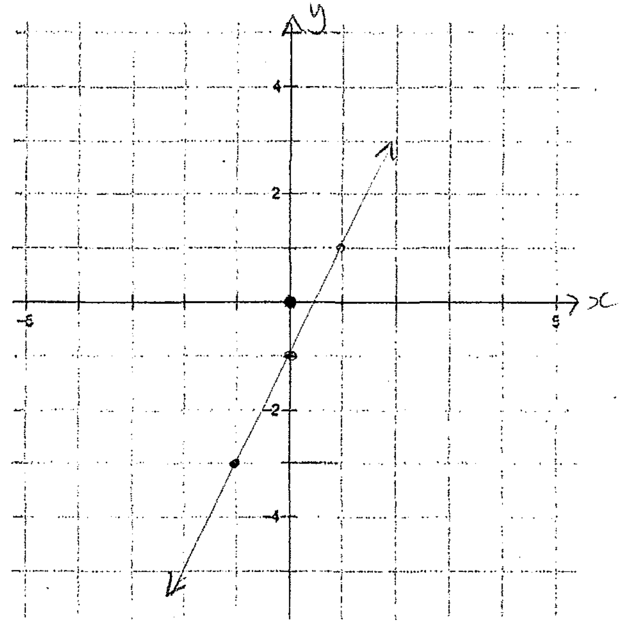
$-y - 1 = 0$

$y = -1$ $\therefore$ y-int is -1.



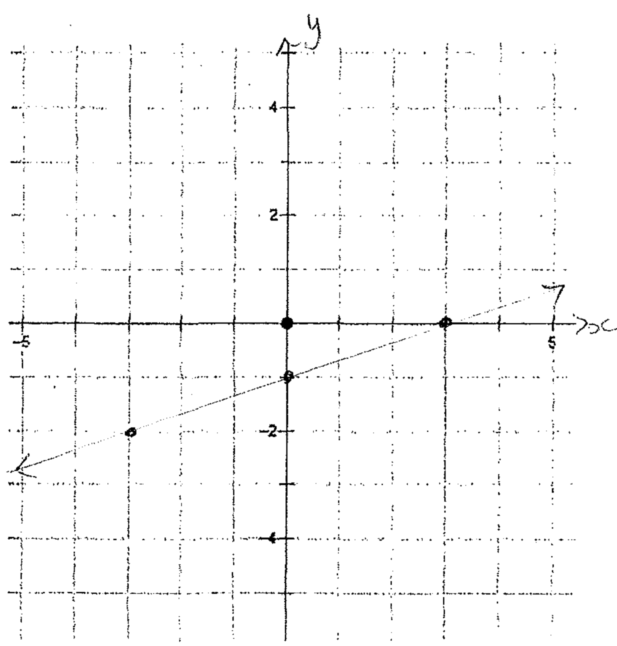
METHOD 3: TABLE OF VALUES (y=mx +b)

X	Y = $2x-1$	POINTS
-1	$= 2(-1)-1$ $= -2-1$ $= -3$	A(-1,-3)
0	$= 2(0)-1$ $= 0-1$ $= -1$	B(0,-1)
1	$= 2(1)-1$ $= 2-1$ $= 1$	C(1,1)



Ex2. Graph $y = \frac{1}{3}x - 1$ using a table of values. (select x-values that are multiple of 3)

x	y
-3	$\frac{1}{3}(-3)-1$ $=-1-1$ $=-2$
0	$\frac{1}{3}(0)-1$ $=0-1$ $=-1$
3	$\frac{1}{3}(3)-1$ $=1-1$ $=0$



10 Academic | **Date:**
Day 2: Review II | **Unit 1: Linear Systems**

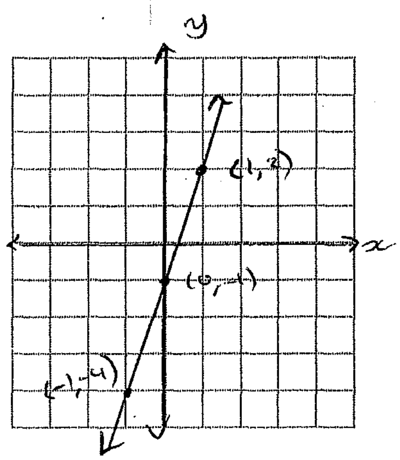
PRACTICE

Graphing

1. Graph each equation using a table of values

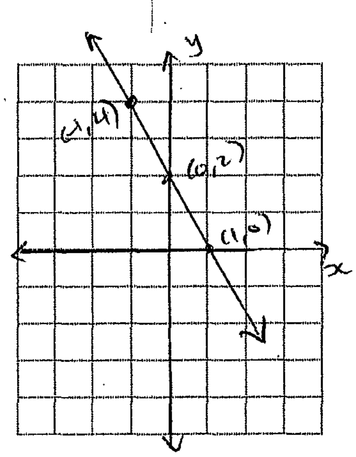
a) $y = 3x - 1$

x	y
-1	-4
0	-1
1	2



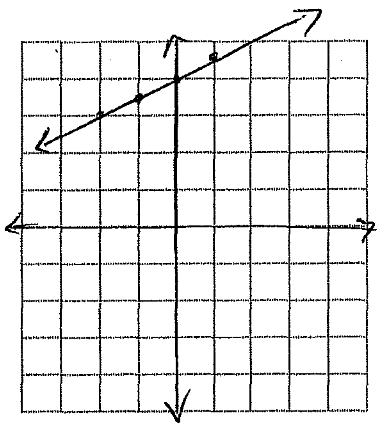
b) $y = -2x + 2$

x	y
-1	4
0	2
1	0



c) $y = ½\,x + 4$

x	y
-1	3.5
0	4
1	4.5

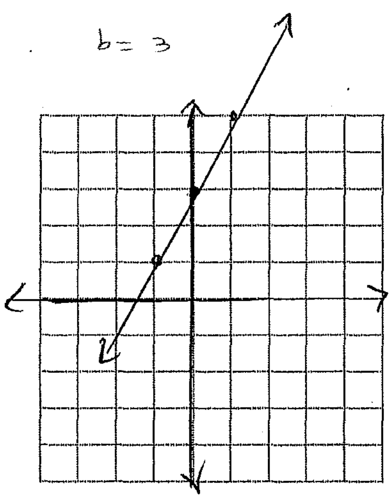


2. Graph each equation using the slope and y-intercept.

a) $y = 2x + 3$

$m = 2$

$b = 3$



$m = 2$

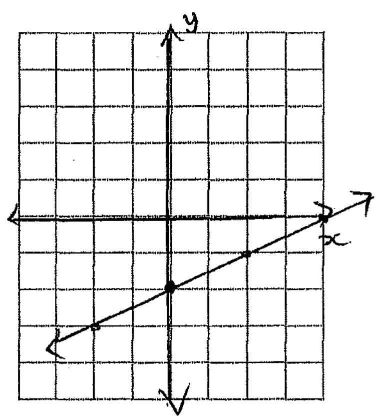
1R UP

OR 1L 2D

b) $y = ½x - 2$

$m = \frac{1}{2}$

$b = -2$



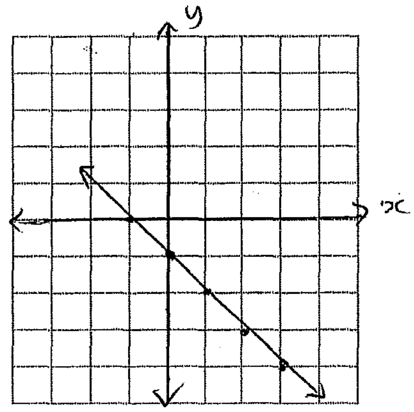
$m = \frac{1}{2}$

2R 1U

c) $x + y + 1 = 0 \Rightarrow y = -x - 1$

$m = -1$

$b = -1$



$m = \frac{-1}{1}$

$\therefore$ 1R 1D

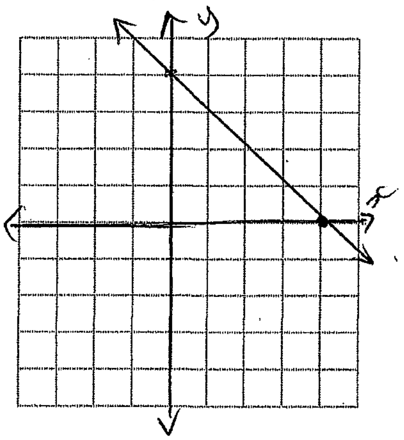
OR 1L 1U

3. Graph each equation by determining the intercepts.

a) $x + y = 4$

x-int: set $y=0$
$x = 4$

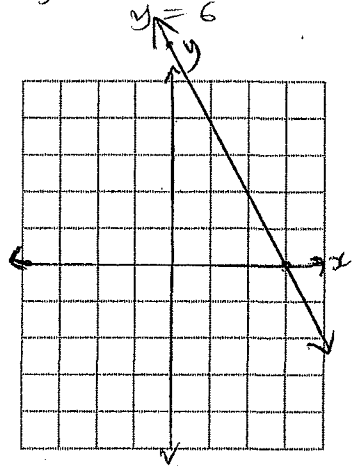
y-int: set $x=0$
$y = 4$



b) $2x + y = 6$

x-int: set $y=0$
$2x = 6$
$x = 3$

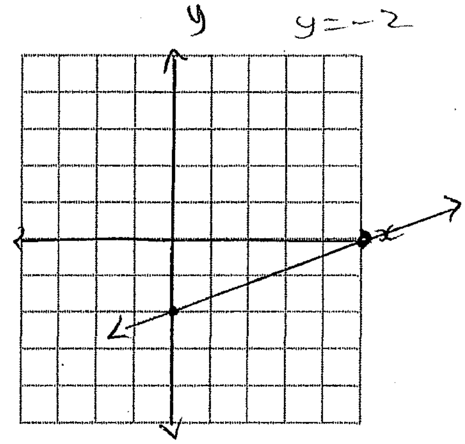
y-int: set $x=0$
$y = 6$



c) $2x - 5y = 10$

x-int: set $y=0$
$2x = 10$
$x = 5$

y-int: set $x=0$
$-5y = 10$
$y = -2$

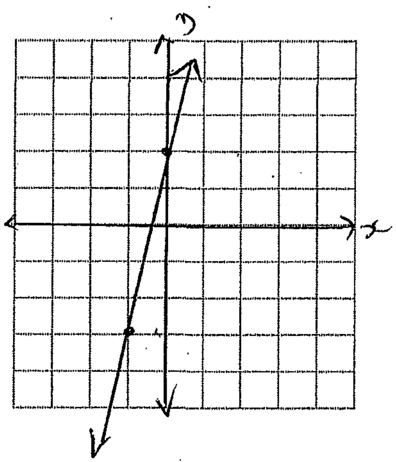


4. Graph each equation using the most suitable method.

a) $y = 5x + 2$

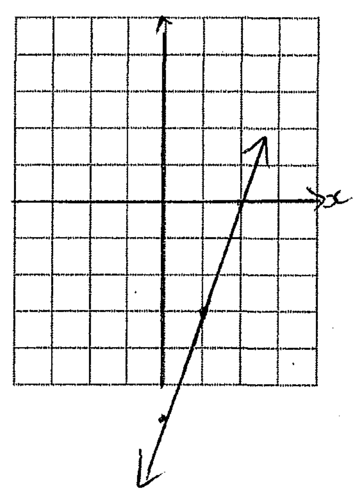
$m = 5$

$b = 2$



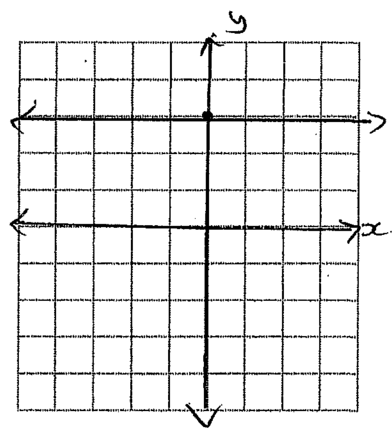
b) $3x - y = 6$

$y = 3x - 6$
$m = 3$
$b = -6$



c) $y = 3$

horizontal line



10 Academic
Day 4: Types of Solutions

Date:
Unit 1: Linear Systems

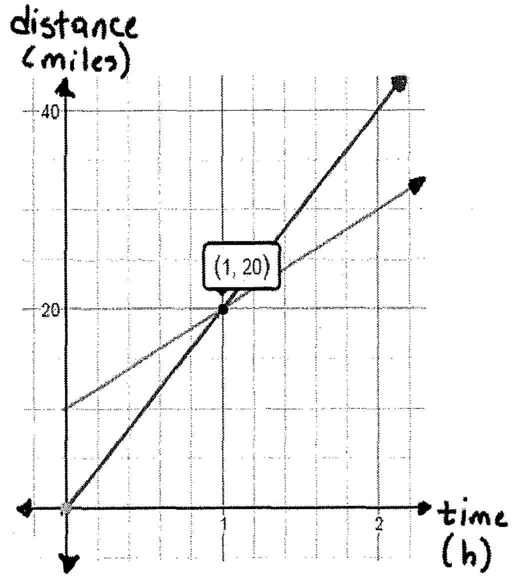
'Intersecting' Lines!?!

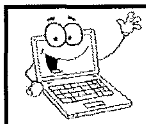
- Things that Make You Go Hmmmm…

Warm-up: Ted and Ned are going to race their dirt bikes. Since Ted is younger, Ned is going to give him a 10 mile head start. Ted travels at 10 mph and Ned travels at 20 mph.
At what time will Ned catch up with Ted?
How far will they have traveled when they meet?

Ted's equation: $y = 10x + 10$
Ned's equation: $y = 20x + 0$

x = time in hours
y = distance in miles





Turn on your **DESMOS**

1. Use the Online Graphing Calculator to graph the two lines. **Sketch** them on the grid to the right. Be sure the point of intersection is showing.

2. Determine the point of intersection of the two lines.

   (1,20)

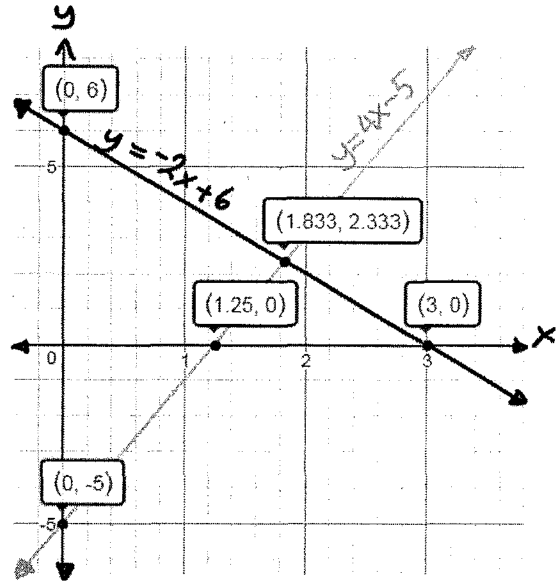
3. What does this point represent in the context of this word problem?

   It's when Ned catches up with Ted

Do 2 lines always intersect in one point? Check it!

Yes, zoom in or out two lines only intersect in one point.

Task 1: One Solution



- Click/ touch on the "x" to delete the equations.
- Change the equation to $y=-2x+6$, and then change the colour of the line to black.
- Change the equation to y = 4x -5, then change the colour of the line to orange.

4. Sketch the two graphs on the grid provided.

5. Why is there one solution to the linear system $\boxed{\begin{matrix} y=-2x+6 \\ y=4x-5 \end{matrix}}$?

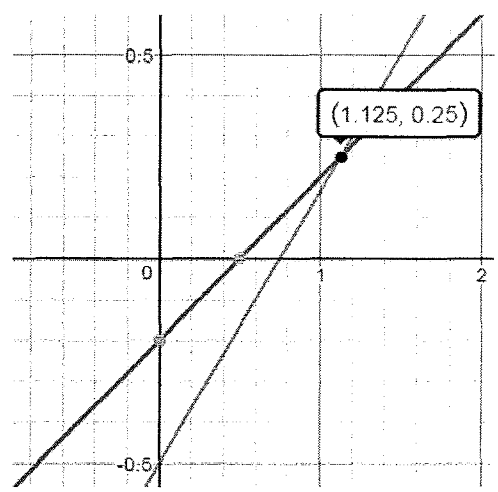
B/c there is only one intersection point.

6. How can you tell by looking at the equations that there will be one solution to the linear system?

They both have different slopes

7. Predict the equation of another line which would have one solution with $y=-2x+6$.

$y=5x-6$ Verify your answer by graphing it on the Desmos.



- Using the **green** line, change the equation to: $2x-5y=1$
- Using the **blue** line, change the equation to: $4x-6y=3$

8. Sketch the two graphs on the grid provided.

9. Why is there one solution to the linear system $\boxed{\begin{matrix} 2x-5y=1 \\ 4x-6y=3 \end{matrix}}$?

1

B/c there is only one intersection point.

10. How can you tell by looking at the equations that there will be one solution to the linear system?

If the first ratios are different, ONE SOLUTION. $\frac{2}{4} \neq \frac{-5}{-6} \neq \frac{1}{3}$

11. Predict the equation of another line which would have one solution with $2x-5y=1$.

$3x-10y=5$ Verify your answer by graphing it on the Desmos.

$6x+10y=2$

$2x-5y=1$

$ax+by=c$

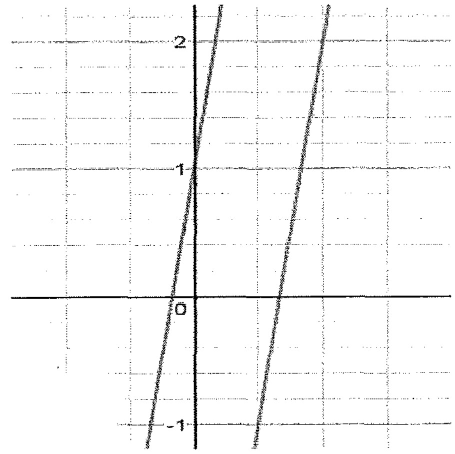
$\frac{2}{a} \neq \frac{-5}{b} \neq \frac{1}{c}$

$a \to 6$, $b \to 10$, $c \to 2$

10 Academic **Date:**
Day 4: Types of Solutions **Unit 1: Linear Systems**

Task 2: No Solution

- Using the green line, change the equation to: $y = 3x - 4$
- Using the blue line, change the equation to: $y = 3x + 1$



12. Sketch the two graphs on the grid provided.

13. Why is there no solution to the linear system $\boxed{\begin{array}{l} y = 3x - 4 \\ y = 3x + 1 \end{array}}$?

Because the lines do not intersect.

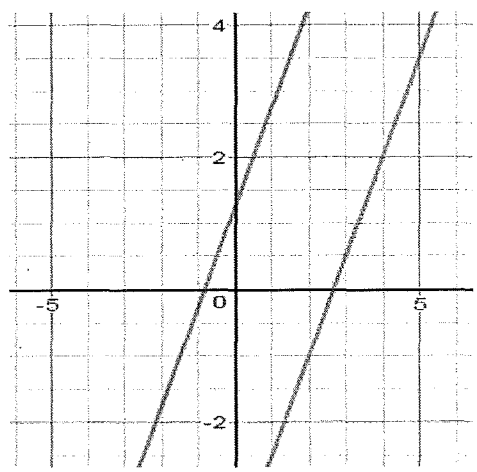
14. How can you tell by looking at the equations that there will not be a solution to the linear system?

They both have the same slope and different y-intercepts.

15. Predict the equation of another line which would have no solution with $y = 3x - 4$.

y = 3x - 1 Verify your answer by graphing it on the Desmos.

- Using the green line, change the equation to: $3x - 2y = 8$
- Using the blue line, change the equation to: $6x - 4y = -5$



16. Sketch the two graphs on the grid provided.

17. Why is there no solution to the linear system $\boxed{\begin{array}{l} 3x - 2y = 8 \\ 6x - 4y = -5 \end{array}}$?

Because two lines are parallel and do not intersect.

18. How can you tell by looking at the equations that there will not be a solution to the linear system?

If the first two Ratios are same but third one is different it is a NO SOLUTION

$\frac{3}{6}, \frac{-2}{-4}, \frac{8}{-5} \Rightarrow \frac{1}{2} = \frac{1}{2} \neq \frac{-8}{5}$

19. Predict the equation of another line which would have no solution with $3x - 2y = 8$.

15x - 10y = 4 Verify your answer by graphing it on the Desmos.

① $3x - 2y = 8$

② $15x - 10y = 4$

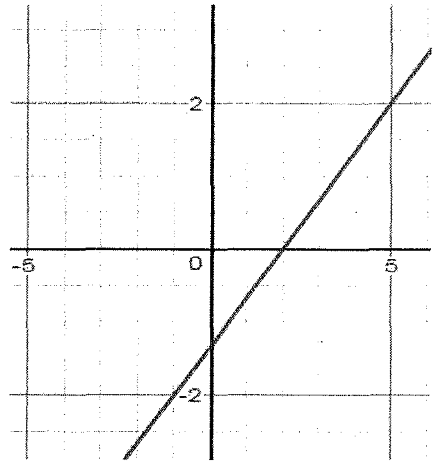
① $3x - 2y = 8$

② $9x - 6y = -8$

$\frac{3}{9}, \frac{-2}{-6}, \frac{8}{-8} \Rightarrow \frac{1}{3} = \frac{1}{3} \neq -1$

Task 3: Many Solutions

- Using the green line, change the equation to: $2x-3y=4$
- Using the blue line, change the equation to: $4x-6y=8$



20. Sketch the two graphs on the grid provided.

21. Why are there multiple solutions to the linear system $\boxed{\begin{matrix} 2x-3y=4 \\ 4x-6y=8 \end{matrix}}$?

Because two lines are coincident. They sit on top of each other.

22. How can you tell by looking at the equations that there will be multiple solutions to the linear system?

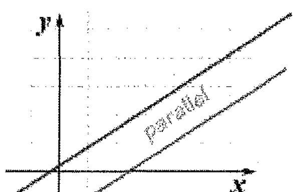
If all of the ratios are the same then it is an Infinite (multiple) solution

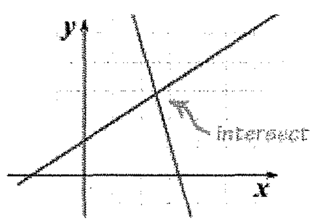
$\frac{2}{4}, \frac{-3}{-6}, \frac{4}{8} \Rightarrow \frac{1}{2}=\frac{1}{2}=\frac{1}{2}$

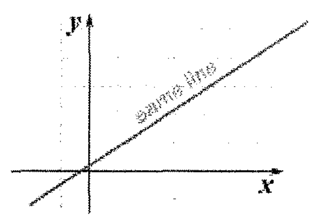
23. Predict the equation of another line which would have multiple solutions with $2x-3y=4$.

6x - 9y = 12 Verify your answer by graphing it on the Desmos.

SUMMARY







$y=mx+b$ →

No Solution	One Solution	∞ Solutions
same slope different y-int	different slope / ratios	same slope same y-int
Only the first two ratios are the same	The first two are different	The ratios are all the same

↗ STD form

Task 4: Practice

24. Determine the number of solutions each linear system has. Justify your decision.

a. $3x-y=5$, $2x+3y=6$ ONE X	b. ×3 $3x+4y=12$ ×3, $-9x-12y=-36$ infinite /	c. $y=3x-5$, $y=4x+6$ ONE X	d. ×5 $2x-3y=10$, $-10x+15y=-15$ NONE //
e. ×0.5 $x+2y=10$, $0.5x+y=8$ NONE ||	f. $3x-5y-2=0$, $4x+5y+2=0$ ONE	g. $y=4x-3$, $y=4x-7$ NONE	h. $x+y=0$, $x-y=0$ ONE

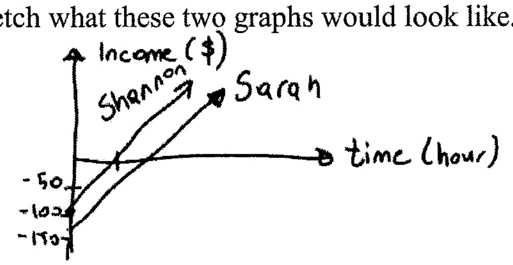
25. Sarah and Shannon mow lawns during the summer to earn money. They both calculated their start-up expenses, operating expenses, and income per hour of mowing. They wrote these equations for their income, *I*, after *h* hours of mowing.
   $I=10.25h-125$ *Sarah*
   $I=10.25h-100$ *Shannon*

   a. What are Shannon's start-up costs? \$100

   b. What does Sarah charge per hour? \$10.25

   c. Will Sarah ever earn as much money as Shannon? Justify your decision.
   Depending on how many hours Sarah might get. If Sarah gets as many hours as Shannon will, then she will make less. On the other hand if Sarah gets more hours, she might earn as much money as Shannon.

   d. Sketch what these two graphs would look like.



26. An air traffic controller is plotting the course of two jets scheduled to land in about 15 minutes. One aircraft is following a path defined by the equation $3x-5y=20$ and the other by the equation $18x=30y+72$. Should the controller alter the paths of either aircraft? Justify your decision.

rearrange

① $3x-5y=20$
② $18x-30y=72$

Two equations are parallel b/c A and B values in the ② equation are 6 times those in the ① equation and C values are different.

OR

Rearrange both equation in $y=mx+b$ form

① $3x-5y=20$ ($-3x$ both sides)
$\frac{-5y}{-5}=\frac{-3x}{-5}+\frac{20}{-5}$
① $\boxed{y=\frac{3}{5}x-4}$

② $18x=30y+72$ (-72 both sides)
$\frac{18x}{30}-\frac{72}{30}=\frac{30y}{30}$
② $\boxed{y=\frac{3}{5}x-\frac{12}{5}}$

∴ They have the same slope and different y-int; therefore, two equations are parallel.