Day 9: Half Angle Formulas

Compound Angle Formulas	Double Angle Formulas	Half Angle Formulas
$sin(a \pm b) = sinacosb \pm sinbcosa$	$sin(2a) = 2sinacosa$	$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{2}}$ *the +/- depends on the quadrant
$cos(a \pm b) = cosacosb \mp sinasinb$	$\cos(2a) = cos^2a - sin^2a$ $= 2cos^2a - 1$ $= 1 - 2sin^2a$	$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+cosx}{2}}$ *the +/- depends on the quadrant
$tan(a \pm b) = \frac{tana \pm tanb}{1 \mp (tana)(tanb)}$	$\tan(2a) = \frac{2tana}{1 - tan^2a}$	$\tan\left(\frac{x}{2}\right) = \frac{1-cosx}{sinx} = \frac{sinx}{1+cosx}$

Example 1: Prove $\frac{1-cosx}{sinx} = \frac{sinx}{1+cosx}$.

$$LS = \frac{1-\cos x}{\sin x} \cdot \frac{1+\cos x}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{\sin x(1+\cos x)} = \frac{\sin^2 x}{\sin x(1+\cos x)} = \frac{\sin x}{1+\cos x} = RS$$

QED

Example 2: Use the half angle formula $\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+cosx}{2}}$ to evaluate $cos15°$.

$$\cos\left(\frac{30}{2}\right) = \sqrt{\frac{1+\cos 30}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

← multiply by 2

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$$= \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

Example 3: Find the exact value of $sin(\frac{\pi}{12})$.

$$= \sin\left(\frac{\frac{\pi}{6}}{2}\right) = +\sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= +\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

Example 4: Use the half angle formulas to find the exact value of the following:

Q2

a) $cos(157.5°) = \cos\left(\frac{315}{2}\right)$

$= -\sqrt{\frac{1+\cos(315)}{2}}$

$= -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = \frac{-\sqrt{2+\sqrt{2}}}{2}$

b) $\tan(\frac{\pi}{8}) = \tan\left(\frac{\frac{\pi}{4}}{2}\right)$

$= \frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}$

$= \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\sqrt{2}}$

$= \sqrt{2}-1$

Practice: Use the half angle formulas to find the exact value of the following.

a. $sin(67.5°)$ $= \sin\left(\frac{135}{2}\right)$ $= \sqrt{\frac{1-\cos 135}{2}}$ $= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$ $= \frac{\sqrt{2+\sqrt{2}}}{2}$	b. $cos(\frac{\pi}{8}) = \cos\left(\frac{\frac{\pi}{4}}{2}\right)$ $= \sqrt{\frac{1+\cos(\frac{\pi}{4})}{2}}$ $= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$ $= \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$	c. $tan(157.5°)$ Q2 $= \tan\left(\frac{315}{2}\right)$ $= \frac{1-\cos 315}{\sin 315}$ $= \frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{-\sqrt{2}}$ $= -\sqrt{2}+1$
d. $sin(\frac{11\pi}{12}) = \sin\frac{\pi}{12}$ Q2 $= \sin\left(\frac{\pi}{6}/2\right)$ $= \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}$ $= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$ $= \frac{\sqrt{2-\sqrt{3}}}{2}$	e. $cos(-112.5°)$ Q3 $= \cos\left(\frac{-225}{2}\right)$ $= -\sqrt{\frac{1+\cos(-225)}{2}}$ $= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$ $= -\sqrt{\frac{2-\sqrt{2}}{4}}$ $= -\frac{\sqrt{2-\sqrt{2}}}{2}$	f. $sin(202.5°)$ Q3 $= -\sin(22.5)$ $= -\sin\left(\frac{45°}{2}\right)$ $= -\sqrt{\frac{1-\cos 45°}{2}}$ $= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$ $= -\sqrt{\frac{2-\sqrt{2}}{4}} = -\frac{\sqrt{2-\sqrt{2}}}{2}$