Day 9: 6.8 Linear Combinations and Spanning Sets

Two **non collinear** vectors, $\vec{u}$ *and* $\vec{v}$ can be written as a **linear combination** $a\vec{u} + b\vec{v}$, where a and b are scalars.

In R^2, any two nonzero, non collinear vectors are said to form a **spanning set** for R^2 if every vector in R^2 can be written as a linear combination of these two vectors. The set of vectors $\{\vec{i}, \vec{j}\}$ forms a spanning set for R^2 since every vector can be written uniquely as a linear combination of these two basis vectors.
Similarly, in R^3, $\{\vec{i}, \vec{j}, \vec{k}\}$ are basis vectors that forma spanning set for R^3.

Ex 1: Write the vector (3, – 5) as a linear combination of the vectors (– 1, 2) and (1, 4).

$[3, -5] = a[-1, 2] + b[1, 4]$

① $3 = -a + b$
② $-5 = 2a + 4b$
use substitution or elimination

①×2 $6 = -2a + 2b$
② $-5 = 2a + 4b$
$1 = 6b$
$b = \frac{1}{6}$

$3 = -a + b$
$3 = -a + \frac{1}{6}$
$-a = 17/6$
$a = -17/6$

$\therefore [3, -5] = -\frac{17}{6}[-1, 2] + \frac{1}{6}[1, 4]$

Ex 2: Determine if each set of vectors is a spanning set for R^2. $\vec{a}$ $\vec{b}$

a) $\{(1, 3), (-2, -1)\}$

Yes. They are not collinear OR

$\vec{a} \neq k\vec{b}$

(and nonzero)

b) $\{(1, 5), (-2, -10)\}$

$[1, 5] = -\frac{1}{2}[-2, -10]$

$\therefore$ No. They span a line in $\mathbb{R}^2$ NOT $\mathbb{R}^2$.

Alternate Solution to Ex. 2:

Note: Any nonzero, non collinear vectors span R^2.

a) If we can find a *k*-value such that $k(1,3) = (-2,-1)$, they are collinear and therefore do not span R^2.

$1k = -2$ $\qquad$ $3k = -1$

$k = -2$ $\qquad$ $k = \frac{-1}{3}$

$\therefore$ No 'k' such that $k(1,3) = (-2,-1)$

b) $k(1,5) = (-2,-10)$

$k = -2$ $\qquad$ $5k = -10$

$k = -2$

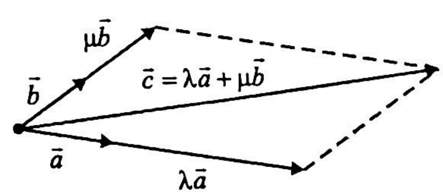
$\therefore$ There is k such that $k(1,5) = (-2,-10)$

Any pair of nonzero, noncollinear vectors will span a plane in R^3. When two or more points or vectors lie on the same plane, they are said to be **coplanar**.

Linear Dependency

Three vectors $\vec{a}, \vec{b}, and\ \vec{c}$ are linear dependent if there exist λ and μ such that $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

Note. In order to be linear dependent the vectors must be coplanar (must belong to the same plan).



Ex 3: Given $\vec{u} = (10, -5, 3)$, $\vec{v} = (8, 2, -1)$. Determine if the given vector lies on the same plane determined by $\vec{u}$ *and* $\vec{v}$.

a) $\vec{x} = (-4, -16, 9)$

$[-4, -16, 9] = a[10, -5, 3] + b[8, 2, -1]$

$-4 = 10a + 8b$ ①

$-16 = -5a + 2b$ ② $\times 4$

$-4 = 10a + 8b$

$-64 = -20a + 8b$

$-$

$60 = 30a$

$\boxed{a = 2}$

$-4 = 10a + 8b$

$-4 = 20 + 8b$

$-24 = 8b$

$\boxed{b = -3}$

Now check the z- coordinate.

$9 = 3a - b$

$9 \stackrel{?}{=} 3(2) - (-3)$

$9 \stackrel{\checkmark}{=} 6 + 3$

$\therefore \vec{x}$ is on the same plane.

b) $\vec{y} = (1, 5, -12)$

$[1, 5, -12] = a[10, -5, 3] + b[8, 2, -1]$

$1 = 10a + 8b$

$5 = -5a + 2b$ $\times 4$:

$1 = 10a + 8b$

$20 = -20a + 8b$

$-19 = 30a$

$\boxed{a = \frac{-19}{30}}$

$1 = 10\left(\frac{-19}{30}\right) + 8b$

$1 + \frac{190}{30} = 8b$

$\boxed{b = \frac{11}{12}}$

check the third equation

$-12 = 3a - b$

$-12 \stackrel{?}{=} 3\left(\frac{-19}{30}\right) - \left(\frac{11}{12}\right)$

$-12 \neq \frac{-169}{60}$

$\therefore \vec{y}$ does not lie on the same plane.