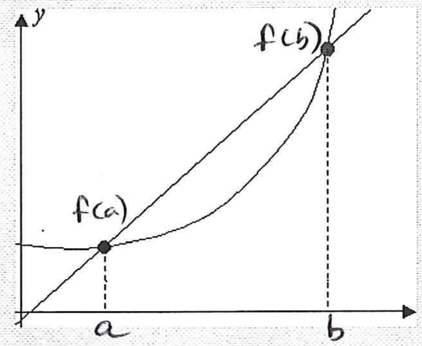
Day 9: 1.5 –Slopes of Secants & Average Rate of Change

Key Terms:

Rate of Change is a measure of how quickly one quantity (the dependent variable) changes with respect to another quantity (the independent variable)

Average Rate of Change is a change that takes place over an interval, or represents the rate of change between two different points.

Secant is a line that connects two points on a curve. The average rate of change between two points corresponds to the **slope of the secant.**



We calculate the average rate of change by calculating: slope between $a \le x \le b$

$$AROC = \frac{f(b)-f(a)}{b-a} \leftarrow \frac{\Delta y}{\Delta x}$$

Example One – A rock is kicked upward from a cliff that is 120 m above the water. The height of the rock above the water $h(t)$, in metres, after time t, in seconds, is modeled by the function, $h(t) = -5t^2 + 10t + 120$

a) Use the equation to determine the end points of each interval, then determine the average rate of change in the height for each time interval. (0, 120) (1, 125)

i. [0, 1]

$$AROC = \frac{h(1)-h(0)}{1-0} = \frac{125-120}{1-0} = 5 \text{ m/s}$$

ii. [1, 2]

$$h(2) = -5(2)^2+10(2)+120 = 120$$

$$AROC = \frac{h(2)-h(1)}{2-1} = \frac{120-125}{1} = -5 \text{ m/s}$$

iii. [2, 3]

$$h(3) = -5(9)+10(3)+120 = 105$$

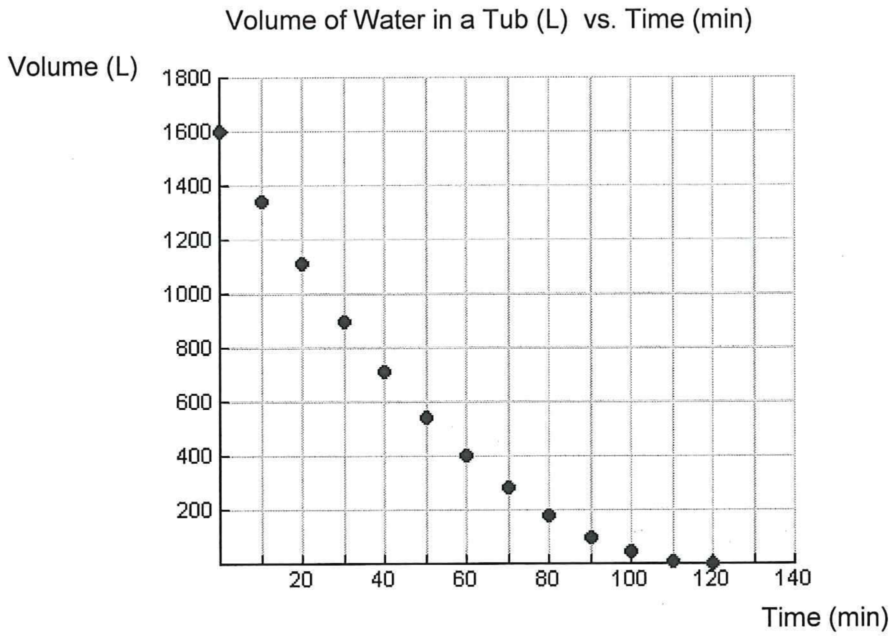
$$AROC = \frac{h(3)-h(2)}{3-2} = \frac{105-120}{1} = -15 \text{ m/s.}$$

b) Describe what the average rate(s) of change you calculated in part a means in this situation.

As x values increases, y values decreases at steeper rate.
(The function is decreasing)

Example Two: Andrew drains the water from a hot tub that holds 1600 L of water. It takes 2 hours for the water to drain completely. The volume V, in litres, of water remaining at various times t, in minutes, is shown in the table below.

Time (min)	Volume (L)
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	10
120	0



a) Calculate the average rate of change in volume during the following time intervals using the table.

i. $0 \leq t \leq 20$ (0, 1600) (20, 1100)

$$AROC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1100 - 1600}{20 - 0} = \frac{-500}{20} = -25 \text{ L/min}$$

ii. $30 \leq t \leq 80$ (30, 900) (80, 200)

$$AROC = \frac{200 - 900}{80 - 30} = \frac{-700}{50} = -14 \text{ L/min}$$

b) Why is the rate of change in volume negative?

The volume of water in the tub is decreasing. (It is being drained)

c) Does the water drain out of the tub at a constant rate?

NO. Initially it drains at much higher rate. (From the graph, it is NOT a linear graph)