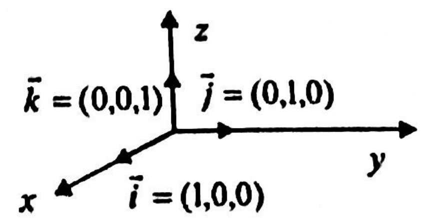
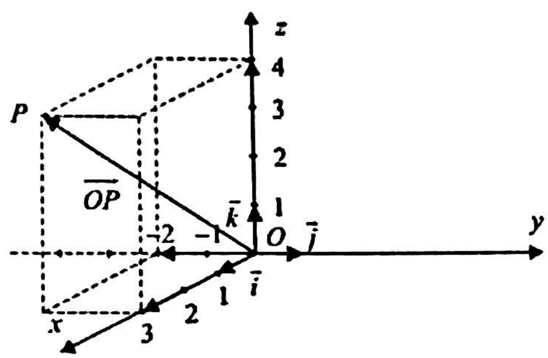
Day 8: 6.7 Operation with Algebraic Vectors in R^3

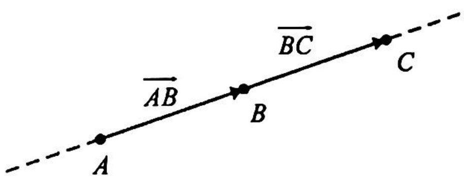
In R^3, the standard basis vectors (unit vectors) are
$\vec{i} = (1,0,0)$, $\vec{j} = (0,1,0)$ *and* $\vec{k} = (0,0,1)$.



If $\vec{OP} = (a,b,c)$ then $\vec{OP} = a\vec{i} + b\vec{j} + c\vec{k}$.



Co-linearity



Three points A, B, and C are collinear if $\overrightarrow{AB} \parallel \overrightarrow{BC}$

Ex 1 : Write the following vectors using the unit vectors $\vec{i}$, $\vec{j}$ *and* $\vec{k}$.

a) $\vec{OP} = (-3,0,2)$

$\vec{OP} = -3\vec{i} + 2\vec{k}$

b) $\vec{OA} = (5,-3,1)$

$\vec{OA} = 5\vec{i} - 3\vec{j} + \vec{k}$

Ex 2: Given $\vec{a} = (-3,2,4)$, $\vec{b} = (1,3,0)$, $\vec{c} = (-2,0,-3)$, determine $2\vec{a} - \vec{b} + 3\vec{c}$

$$2\vec{a} - \vec{b} + 3\vec{c} = 2[-3,2,4] - [1,3,0] + 3[-2,0,-3]$$

$$= [-6,4,8] + [-1,-3,0] + [-6,0,-9]$$

$$= [-13,+1,-1] \quad \text{or} \quad -13\vec{i} + \vec{j} - \vec{k}$$

If $A(x_1, y_1, z_1)$ *and* $B(x_2, y_2, z_2)$ are two points in R^3, then

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \quad \textit{head minus tail}$$

$$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

and $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Example: If $A(3,-5,2)$ and $B(1,-4,-6)$ are points in R^3, determine:

a) $|\vec{OA}|$

$\overrightarrow{OA} = [3, -5, 2]$

$|\overrightarrow{OA}| = \sqrt{3^2 + (-5)^2 + 2^2}$

$= \sqrt{9 + 25 + 4}$

$= \sqrt{38}$

b) $A\vec{B}$

$\overrightarrow{AB} = [1-3, -4-(-5), -6-2]$

$= [-2, 1, -8]$

c) $B\vec{A}$

$\overrightarrow{BA} = -\overrightarrow{AB}$

$= -[-2, 1, -8]$

$= [2, -1, 8]$

d) $|A\vec{B}|$

$|\overrightarrow{AB}| = \sqrt{(-2)^2 + (1)^2 + (-8)^2}$

$= \sqrt{4 + 1 + 64}$

$= \sqrt{69}$