Day 8: 7.2– Solving Logarithmic Equations

Warm Up - Solve for x. Identify any extraneous roots.

$2^{2x} - 7(2)^x = 8$

$(2^x)^2 - 7(2^x) - 8 = 0$

$(2^x - 8)(2^x + 1) = 0$

$2^x = 8$ $\quad$ $2^x = -1$

$x = 3$ $\quad$ NO SOLN.

We can also solve logarithmic equations. To solve a logarithmic equation:

1. Use strategies to **isolate the variable**:
   - Logarithm Laws (Change of Base, Power, Product, Quotient)
   - Change between exponential and logarithmic form
   - Write both sides as logs with the same base & equate logs
     ie: if $\log a = \log b$ then $a = b$
2. Solve for x, factor if necessary
   - Check for extraneous roots - *need to check with original equation*

EX 1 – Solve for x. Identify any extraneous roots.

a) $\log_5(2x - 3) = 2$

$5^2 = 2x - 3$

$2x - 3 = 25$

$2x = 28$

$x = 14$

(checked.. works)

$\log_5(2(14) - 3)$

$= \log_5(25)$

$= 2$

b) $\log(x - 1) - 1 = -\log(x + 2)$

$\log(x-1) + \log(x+2) = 1$

$\log(x-1)(x+2) = 1$

$10^1 = (x-1)(x+2)$

$x^2 + x - 2 - 10 = 0$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4 \rightarrow$ inadmissible

$\boxed{x = 3}$

a) $\log \sqrt[3]{x^2+48x} = \frac{2}{3}$

$\log (x^2+48x)^{\frac{1}{3}} = \frac{2}{3}$

$\frac{1}{3}\log(x^2+48x) = \frac{2}{3}$

$\log(x^2+48x) = 2$

$x^2+48x = 100$

$x^2+48x-100 = 0$

$(x+50)(x-2) = 0$

$x = -50 \qquad x = 2$

both solutions work $\therefore x = \{-50, 2\}$

b) $\log x^4 - \log 3 = \log(3x^2)$

$\log \frac{x^4}{3} = \log(3x^2)$

$\therefore \frac{x^4}{3} = 3x^2$

$x^4 - 9x^2 = 0$

$x^2(x^2-9) = 0 \Rightarrow (x^2)(x-3)(x+3) = 0$

$x = \{0, +3, -3\}$

↳ all answers work.