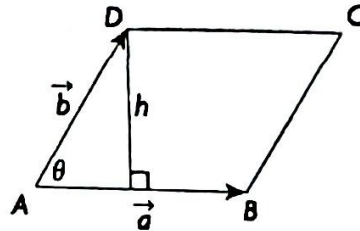


Day 7: 7.7 Applications of Cross and Dot Product

Area of a Parallelogram:

$$A = bh$$

$$\begin{aligned} \text{Area} &= |\vec{a}|h \\ &= |\vec{a}|(|\vec{b}|\sin\theta) \\ &= |\vec{a}||\vec{b}|\sin\theta \end{aligned}$$

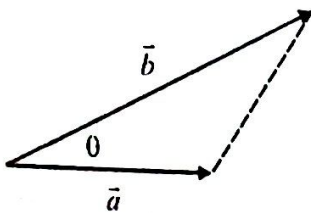


Ex1: Determine the area of the parallelogram determined by the vectors $\vec{p} = (-1, 5, 6)$ and $\vec{q} = (2, 3, -1)$

$$\begin{aligned} A_{\square} &= |\vec{p} \times \vec{q}| \\ &= |(-23, 11, -13)| \\ &= \sqrt{(-23)^2 + (11)^2 + (-13)^2} \\ &= \sqrt{819} = 3\sqrt{91} \text{ units}^2 \end{aligned}$$

$$\begin{array}{r} 5 \quad 3 \quad (-5-18) \\ 6 \quad -1 \quad (12-1) \\ -1 \quad 2 \quad (-3-10) \\ 5 \quad 3 \end{array}$$

Ex2: Find the Area of the triangle ABC whose vertices are A(1, 3, 5), B(-2, -3, 4) and C(0, 3, -1).



$$\begin{aligned} \vec{a} &= \vec{AB} = (-3, -6, -1) \\ \vec{b} &= \vec{AC} = (-1, 0, -6) \end{aligned}$$

$$\begin{aligned} A_{\Delta} &= \frac{A_{\square}}{2} = \frac{|\vec{a} \times \vec{b}|}{2} \\ &= \frac{|(36, -17, -6)|}{2} \end{aligned}$$

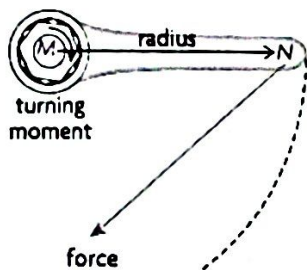
$$\begin{array}{r} -6 \quad 0 \\ -1 \quad -6 \\ -3 \quad -1 \\ -6 \quad 0 \end{array}$$

$$= \frac{\sqrt{36^2 + (-17)^2 + (-6)^2}}{2} = \frac{\sqrt{1621}}{2} \approx 20.13 \text{ units}^2$$

Torque is a vector quantity measured in Newton-metres (N-m) (or in joules (J)). Force causes an object to turn causing an angular rather than a linear displacement. The torque is caused by a force defined as the cross product.

$Torque = \vec{r} \times \vec{F}$ or $Torque = |\vec{r}| |\vec{F}| \sin \theta$ where \vec{F} is the applied force, $|\vec{F}|$ in Newtons and \vec{r} is the vector acting on the axis of rotation, $|\vec{r}|$ in meters, θ is the angle between \vec{r} and

Ex3: A force of 90N is applied to a wrench 15 cm long at 70° to the handle. Determine the torque.



NOTE: $r = 0.15 \text{ m}$

$$\begin{aligned}
 \text{Torque} &= |\vec{r}| |\vec{F}| \sin \theta \\
 &= (0.15) (90) \sin 70 \\
 &= 12.7 \text{ Nm}
 \end{aligned}$$

