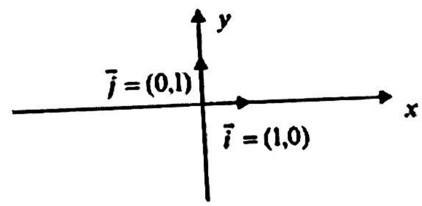
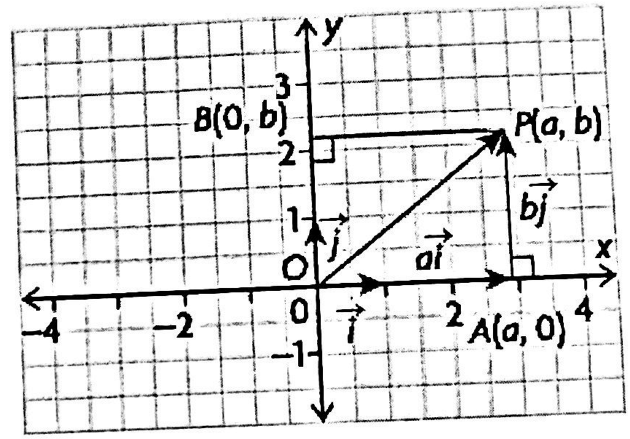
Day 7: 6.6 Operation with Algebraic Vectors in R^2

Recall: The position vector $\vec{OP}$ can be represented in component form, $\vec{OP} = (a,b)$.



Unit vectors with magnitude 1 lie along the positive x- and y-axes and are represented as $\vec{i} = (1,0)$ *and* $\vec{j} = (0,1)$ in R^2. $\vec{i}$ *and* $\vec{j}$ are called the standard basis vectors in R^2. Every vector in R^2, given in terms of its components, can also be written uniquely in terms of $\vec{i}$ *and* $\vec{j}$.



If $\vec{OP} = (a,b)$ *then* $\vec{OP} = a\vec{i} + b\vec{j}$

and $|\vec{OP}| = \sqrt{a^2 + b^2}$

Ex:

1. Write the following vectors using the unit vectors $\vec{i}$ *and* $\vec{j}$.

a) $\vec{OP} = (-2, 3)$

$\vec{OP} = -2\vec{i} + 3\vec{j}$

b) $\vec{OA} = (0, -5)$

$\vec{OA} = -5\vec{j}$

2. Write the following vectors in component form.

a) $\vec{OQ} = 2\vec{i} - 4\vec{j}$

$\vec{OQ} = [2, -4]$

b) $\vec{OS} = 3\vec{i}$

$\vec{OS} = [3, 0]$

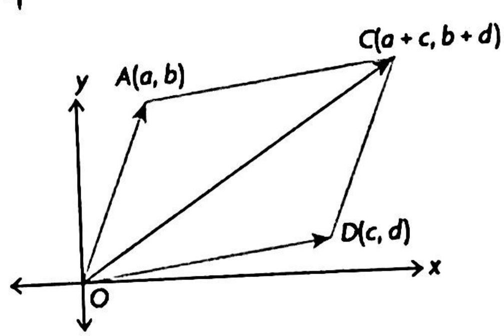
3. Find the vector components and the magnitude for $\vec{a} = (-3, -5)$

$\vec{a} = -3\vec{i} - 5\vec{j}$

$|\vec{a}| = \sqrt{(-3)^2 + (-5)^2}$

$= \sqrt{34}$

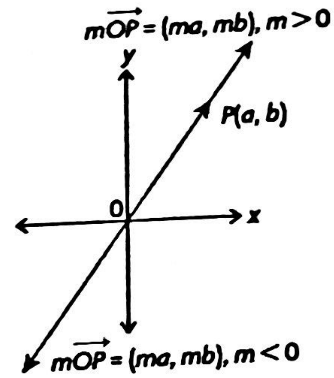
To determine the sum of two algebraic vectors, add their corresponding x- and y-components.



$[a, b] + [c, d] = [a+c, b+d]$

To multiply an algebraic vector by a scalar, multiply both x- and y-components by the scalar.

$m\vec{OP} = (ma, mb)$



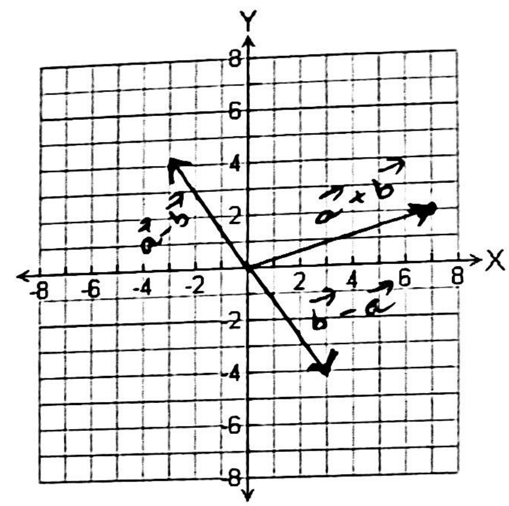
Ex: Given $\vec{a} = \vec{OA} = (2,3)$ and $\vec{b} = \vec{OB} = (5,-1)$, determine the components of $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$, and $\vec{b}-\vec{a}$. Illustrate each of these vectors in R^2.

$\vec{a}+\vec{b} = [2,3] + [5,-1] = [7,2]$

$\vec{a}-\vec{b} = [2,3] - [5,-1] = [-3,4]$

$\vec{b}-\vec{a} = -[-3,4] = [3,-4]$

$\hookrightarrow \vec{b}-\vec{a} = -(\vec{a}-\vec{b})$



> If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points in R^2, then
>
> $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ *head minus tail*
>
> $\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$
>
> and $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex: Given A(5, – 3) and B(8, 5) are points in R^2, determine $A\vec{B}$ and $|A\vec{B}|$.

$\vec{AB} = [8,5] - [5,-3]$

$= [3,8]$ Logical way: from 5 to 8 is 3
-3 to 5 is 8

$|\vec{AB}| = \sqrt{3^2+8^2}$

$= \sqrt{73}.$

Ex: If $\vec{a} = (2, -4)$, $\vec{b} = (-6, 3)$ and $\vec{c} = (3, 5)$, calculate $\left|\frac{1}{2}\vec{a} - \vec{b} + 2\vec{c}\right|$

$$\frac{1}{2}\vec{a} - \vec{b} + 2\vec{c} = \frac{1}{2}[2,-4] - [-6,3] + 2[3,5]$$

$$= [1,-2] + [6,-3] + [6,10]$$

$$= [13, 5]$$

$$\left|\frac{1}{2}\vec{a} - \vec{b} + 2\vec{c}\right| = \sqrt{13^2 + 5^2}$$

$$= \sqrt{169 + 25}$$

$$= \sqrt{194}$$

Ex: Given $A(-2,3)$, $B(-1,-2)$, and $D(4,2)$, find the point C such that the polygon $ABCD$ is a parallelogram.

$\vec{AB} = \vec{DC}$ Let $C(x,y)$

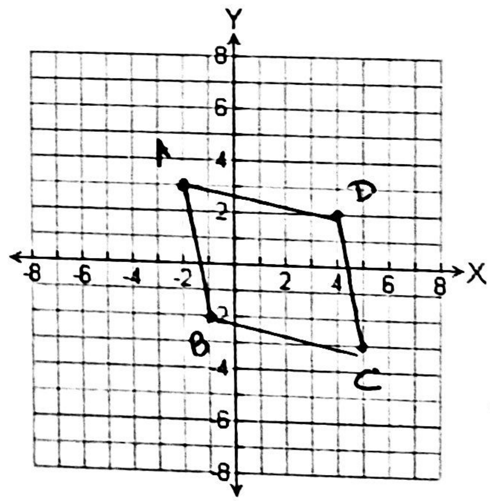
$$[-1-(-2), -2-3] = [x-4, y-2]$$

$$[1, -5] = [x-4, y-2]$$

$x - 4 = 1$ $\quad\quad$ $-5 = y - 2$

$x = 5$ $\quad\quad$ $y = -3$

$\therefore C(5,-3)$



We could also have done $\vec{AD} = \vec{BC}$