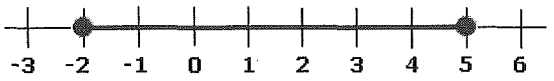
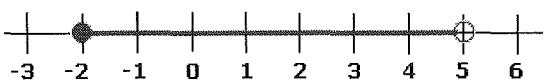
Day 7: 2.5 *Solving Polynomial Inequalities*

Recall: An **inequality** is a mathematical statement that contains one of the following symbols $<, \leq, >, \geq$ or $\neq$.

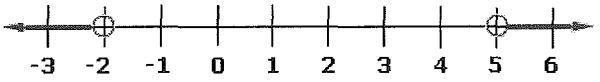
Warm-up: Write an inequality that corresponds to the values of x shown on each number line.
Note: a solid dot indicates that the value is included in the interval, a hallow dot indicates it is excluded

a) 

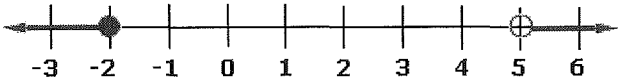
$-2 \leq x \leq 5$ OR $x \in [-2, 5]$

b) 

$-2 \leq x < 5$ OR $x \in [-2, 5)$

c) 

$x > 5$ or $x < -2$ OR $x \in (-5, \infty) \cup (-\infty, -2)$

d) 

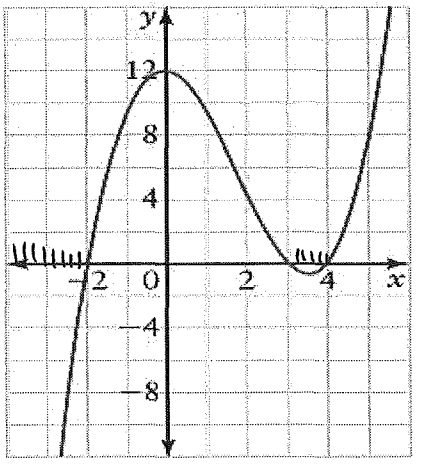
$x > 5$ or $x \leq -2$

OR $x \in (-\infty, -2] \cup (5, \infty)$

When we **"solve"** an inequality we are finding the values of x that **make the inequality true.**

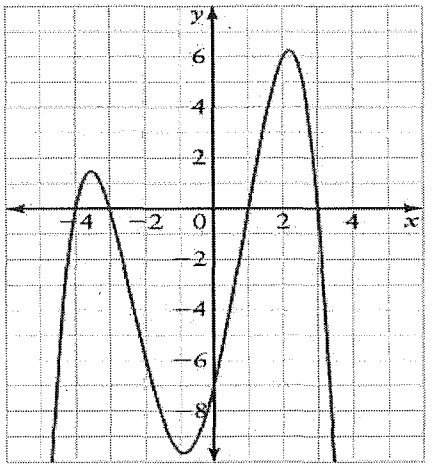
Example One - Solve inequalities given a graph

a) Solve $f(x) \leq 0$



$x \leq -2$ or $3 \leq x \leq 4$

b) Solve $f(x) > 0$



$-2 < x < 3$ or $x > 4$

In general, to solve a polynomial inequality algebraically:

1. Gather all terms on LHS and factor the inequality
2. State the zeros
3. Draw an **interval table**, with the zeros as your intervals
4. Test values within each interval to determine if the solution is (+) or (-)
5. Determine whether each interval satisfies the inequality
6. State the solution

Example Two– Solve the following inequality using **an interval table:** $(x+3)(2x-3)>0$ $x=-3, \frac{3}{2}$

	$(-\infty,-3)$	$(-3,3/2)$	$(3/2,\infty)$
$x+3$	−	+	+
$2x-3$	−	−	+
$f(x)$	+	−	+

$\therefore x \in (-\infty,-3) \cup (\frac{3}{2},\infty)$

Instead of creating a table, graph or a number line may also be used.

Example Three – Solve the following inequality using an interval table: $5x^3-12x^2-11x \le -6$

$\underbrace{5x^3-12x^2-11x+6}_{P(x)} \le 0 \Rightarrow$ FACTOR

$P(1)\neq 0$ $P(-1)\neq 0$

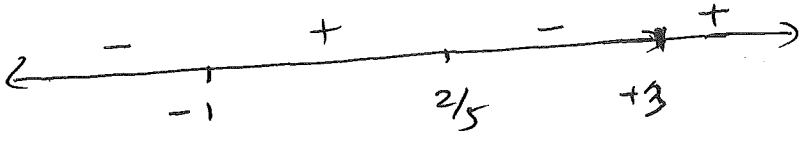
$P(2)\neq 0$ $P(-2)\neq 0$

$P(3)=0 \Rightarrow x-3$ is a factor

3	5	−12	−11	6
		15	9	−6
	5	3	−2	0

$(x-3)(5x^2+3x-2) \le 0$

$(x-3)(5x-2)(x+1) \le 0$



$\therefore x \in (-\infty,-1] \cup [\frac{2}{5},3]$

↓ included since ≤ 0

$5x^2+3x-2 = 5x^2+5x-2x-2$

$= 5x(x+1)-2(x+1)$

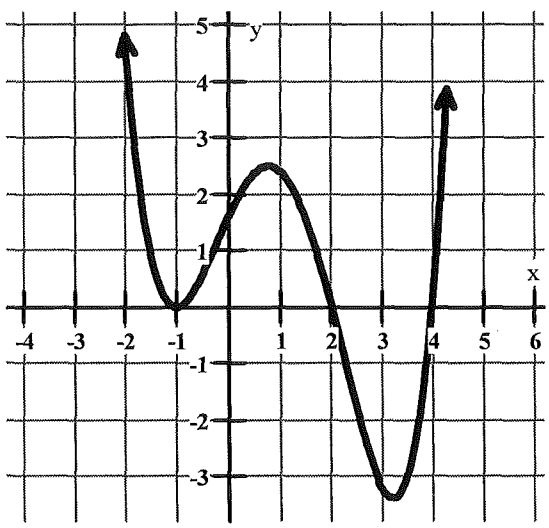
$= (5x-2)(x+1)$

Practice Graphical Questions:

Solve the following inequality from the graphs given:

a) $f(x) \leq 0$

$x = -1$ or $2 \leq x \leq 4$

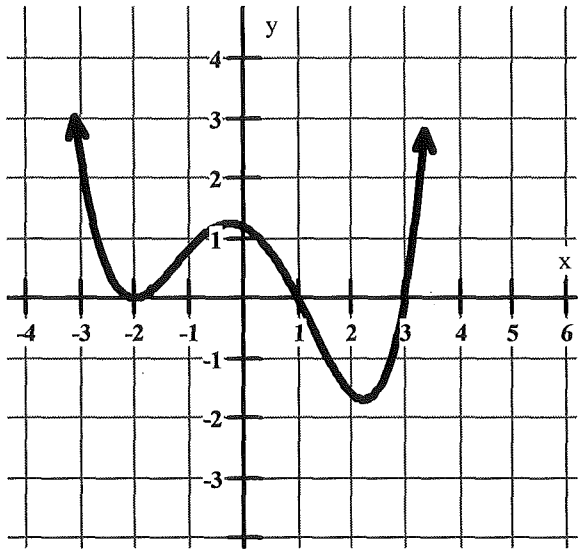


In interval notion:

$x = -1$ or $x \in [2, 4]$

b) $f(x) > 0$

$x < -2$ or $-2 < x < 1$ or $x > 3$



$x \in (-\infty, -2) \cup (-2, 1) \cup (3, \infty)$