# *Day 7: 1.3 – Equations and Graphs of Polynomial Functions*

***Continued***

Note:

***Even functions:***

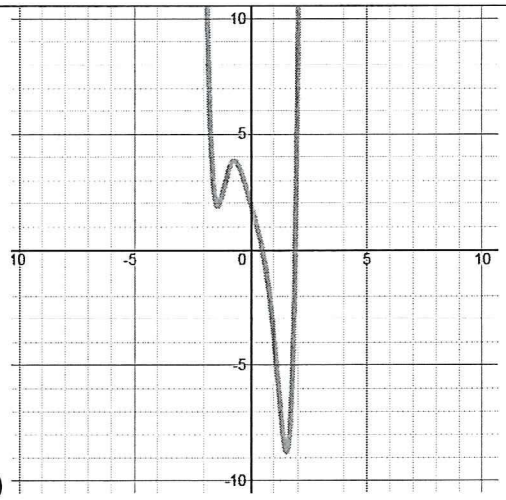
- An even function algebraically satisfies the property $f(-x) = f(x)$ and is symmetric about the y-axis
- An even-degree polynomial function is an even function if the exponent of each term of the equation is even

***Odd functions:***

- An odd function algebraically satisfies the property $f(-x) = -f(x)$ and is rotationally symmetric about the origin
- An odd-degree polynomial function is an odd function if the exponent of each term is odd.

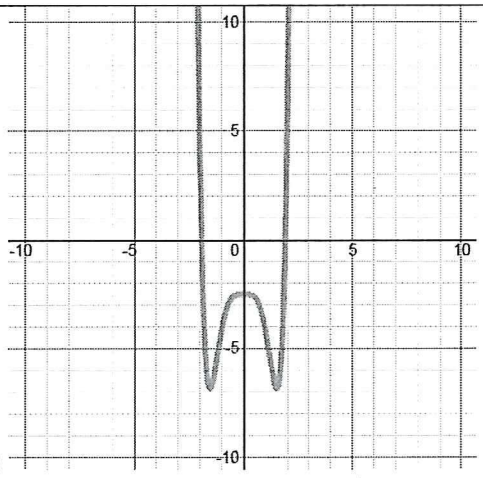
***Example One:*** Determine if the following functions are even, odd or neither from the following graphs:

a)



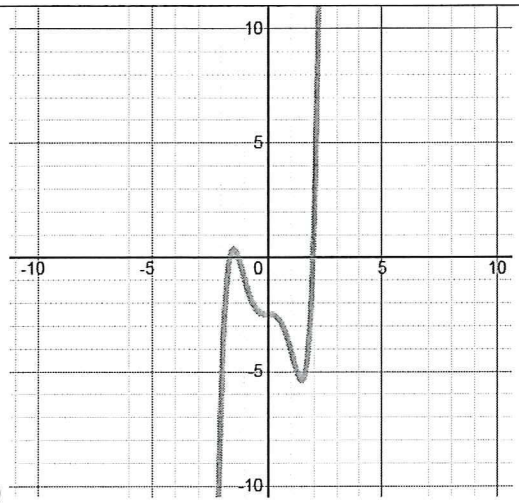

NEITHER

b)



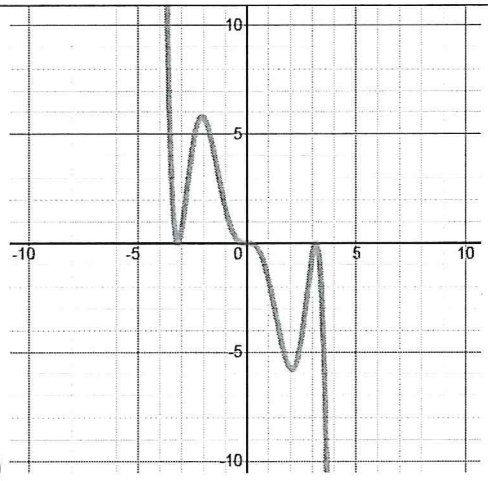

EVEN

c)




NEITHER

d)




ODD.

***Example Two:*** Determine if the following functions are even, odd or neither algebraical

a) $f(x) = 2x^4 - 5x^2 + 4$

$f(-x) = 2(-x)^4 - 5(-x)^2 + 4$

$= 2x^4 - 5x^2 + 4$

$\therefore f(x) = f(-x)$

It is an EVEN FUNCTION.

b) $g(x) = -3x^5 + 9x^3 + 2x$

$g(-x) = -3(-x)^5 + 9(-x)^3 + 2(-x)$

$= +3x^5 - 9x^3 - 2x$

NOT EVEN since $g(x) \neq g(-x)$

$-g(x) = -(-3x^5 + 9x^3 + 2x)$

$= 3x^5 - 9x^3 - 2x$

Since $g(-x) = -g(x)$

$g(x)$ is an ODD function.

c) $h(x) = -x^3 + 2x - 4$

$h(-x) = -(-x)^3 + 2(-x) - 4$

$= x^3 - 2x - 4$

$-h(x) = -(-x^3 + 2x - 4)$

$= x^3 - 2x + 4$

$h(x) \neq h(-x)$ and $-h(x) \neq h(-x)$ $\therefore$ NEITHER

d) $f(x) = x(x-2)^2(x+2)^2$

$f(-x) = -x(-x-2)^2(-x+2)^2$

$= -x(-1(x+2))^2(-(x-2))^2$

$= -x(x+2)^2(x-2)^2$

$f(x) \neq f(-x)$ NOT EVEN.

$-f(x) = -(x)(x-2)^2(x+2)^2$

Since $f(-x) = -f(x)$

$f(x)$ is ODD.

e) $f(x) = -(x+1)^2(x-2)(x+2)$

$f(-x) = -(-x+1)^2(-x-2)(-x+2)$

$= -(-(x-1))^2(-1)(x+2)(-1)(x-2)$

$= -(x-1)^2(x+2)(x-2)$

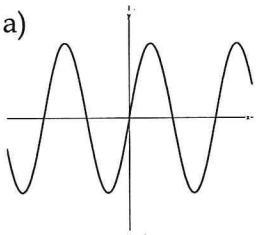
$\therefore$ NOT EVEN

$-f(x) = -(x+1)^2(x-2)(x+2)$

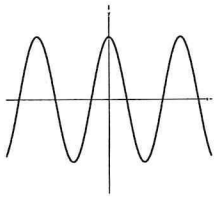
$f(-x) \neq -f(x)$ $\therefore$ NEITHER.

# Even and Odd Worksheet

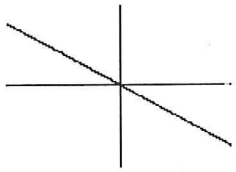
1. Referring to the graph of the function below, state whether the function is even, odd or neither. Justify your answer.

a) 

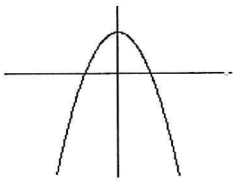
odd

b) 

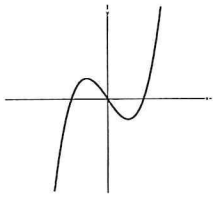
even

c) 

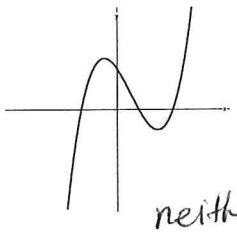
odd

d) 

even

e) 

odd

f) 

neither

2. By inspection, determine whether the function is even, odd, or neither.

a) $f(x)=2x^3$ odd b) $f(x)=x^3-4$ odd c) $f(x)=1-x^2$ even

3. Determine algebraically whether the given function is even, odd or neither.

a) $f(x)=-2x^3-4x$ odd b) $f(x)=-2x^3-3x^2-4x+4$ neither

c) $h(x)=-5x$ odd d) $f(x)=-|x|$ even

e) $f(t)=t^4-6t^2+5$ even f) $h(x)=\sqrt{x^2-5}$ neither

g) $f(x)=\dfrac{x^2+2}{x-7}$ neither h) $f(x)=\dfrac{x^3+x}{x^3-x}$ odd

i) $f(x)=x^2-|x|$ even j) $f(x)=|2x^3|$ even.

**Answers:**

1. a) odd b) even c) odd d) even e) odd f) neither

2. a) odd b) neither c) even

3. a) odd b) neither c) odd d) even e) even f) even g) neither h) even i) even j) even