Day 7/8– Trigonometric Identities

- A **Trig Identity** is a mathematical expression that is TRUE for all angles
- To prove (**algebraically**) a trig identity, work with the more complex side and try to transform it into the other side so that **L.S. = R.S.**
- There are no steps to prove a trig identity. There are various strategies to use, and the more you **practice**, the better you will be at proving them.

Strategies:

- Start from the more complicated side
- Start from the side with a double angle – $\sin 2\theta$ or $\cos 2\theta$
- Write the expression in terms of sin and cos *(Quotient, Reciprocal Identities)*
- Get a common denominator *(when working with adding/subtracting fractions)*
- Factor and/or expand and simplify the expression (ie - $\cos x \sin x + \cos x = \cos x(\sin x + 1)$)
- Apply Pythagorean identities if there are squared terms
- Write the entire equation in terms of one trig function.
- Multiply by the conjugate of an expression ($\frac{\cos x}{1-\sin x}$ would be multiplied by $\frac{1+\sin x}{1+\sin x}$)
- Apply compound angle formulae

Notation to know:

$\sin^2\theta =$ $[\sin\theta]^2$

$2\sin\theta =$ $(2)(\sin\theta)$

$\cos 2\theta =$ $\cos(2\theta)$

EX 1 – Prove the following trig identities. Show all steps.

a) $\cot\theta \sin\theta = \cos\theta$

$LS = \cot\theta \sin\theta$

$= \frac{\cos\theta}{\sin\theta} \sin\theta$

$= \cos\theta$

$= RS$ Q.E.D. ▨

b) $\sin^2\theta - \sin^4\theta = \cos^2\theta - \cos^4\theta$

$LS = \sin^2\theta - \sin^4\theta$

$= \sin^2\theta(1 - \sin^2\theta)$

$= \sin^2\theta \cos^2\theta$

$= (1 - \cos^2\theta)\cos^2\theta$

$= \cos^2\theta - \cos^4\theta$

$= RS.$

Q.E.D ▨

c) $\sin\theta = \csc\theta - \cos\theta \cot\theta$

$RS = \frac{1}{\sin\theta} - \cos\theta \frac{\cos\theta}{\sin\theta}$

$= \frac{1 - \cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta}$

$= \sin\theta = LS$

Q.E.D. ▨

d) $\frac{\csc\theta}{2\cos\theta} = \csc 2\theta$

$LS = \frac{\csc\theta}{2\cos\theta} = \frac{1}{2\sin\theta\cos\theta}$

$= \frac{1}{\sin 2\theta}$

$= \csc 2\theta = RS$

Q.E.D ▨

e) $2\cos y \cos x = \cos(x+y) + \cos(x-y)$	f) $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{2}{\sin\theta}$

e)

$RS = \cos(x+y) + \cos(x-y)$

$= \cos x \cos y - \cancel{\sin x} \sin y$

$+ \cos x \cos y + \sin x \sin y$

$= 2\cos x \cos y.$

$= LS$

QED.

f)

$LS = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \sin^2\theta + 1 + 2\cos\theta + \cos^2\theta$

$= \dfrac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$

$= \dfrac{2}{\sin\theta}$

$= RS$

QED ☒

g) $\cos(x+y)\cos(x-y)=\cos^2 x+\cos^2 y-1$

$LS = (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

DOS

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x \cos^2 y - (1-\cos^2 x)(1-\cos^2 y)$

$= \cos^2 x \cos^2 y - [1 - \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y]$

$= \cos^2 x + \cos^2 y - 1$

$= RS$ QED.

More Trig Identity Practice with Double Angles

<table>
<tr>
<td>

1) $\tan 2x - \sin 2x = 2\tan 2x \sin^2 x$

$LS = \dfrac{\sin 2x}{\cos 2x} - \sin 2x$

$= \sin 2x \left[\dfrac{1}{\cos 2x} - 1 \right]$

$= \sin 2x \left[\dfrac{1 - \cos 2x}{\cos 2x} \right]$

$= \dfrac{\sin 2x}{\cos 2x} \left[1 - (1 - 2\sin^2 x) \right]$

$= \tan 2x \,(2\sin^2 x)$

$= RS.$ Q.E.D.

</td>
<td>

2) $\cos 2x = \cos^2 x - \sin^2 x$

$LS = \cos 2x$

$= \cos(x + x)$

$= \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

$= RS.$

Q.E.D

</td>
</tr>
<tr>
<td>

3) $\sin 2\theta = 2\cot\theta \ast \sin^2\theta$

$RS = 2\cot\theta \sin^2\theta$

$= 2\dfrac{\cos\theta}{\sin\theta} \cdot \sin^2\theta$

$= 2\sin\theta\cos\theta$

$= \sin 2\theta$

$= LS$

Q.E.D

</td>
<td>

4) $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$RS = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= \dfrac{1 - \tan^2\theta}{\sec^2\theta}$

$= \cos^2\theta \left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\right)$

$= \cos^2\theta - \sin^2\theta$

$= \cos 2\theta$

</td>
</tr>
</table>

5) $\frac{2}{1+\cos 2\theta} = \sec^2\theta$

$LS = \frac{2}{1+\cos 2\theta}$

$= \frac{2}{1+2\cos^2\theta - 1}$

$= \frac{2}{2\cos^2\theta}$

$= \sec^2\theta$

$= RS$ Q.E.D ▨

6) $\frac{1-\cos 2\theta}{2} = \sin^2\theta$

$LS = \frac{1-\cos 2\theta}{2}$

$= \frac{1-(1-2\sin^2\theta)}{2}$

$= \frac{2\sin^2\theta}{2}$

$= \sin^2\theta$

$= RS$ Q.E.D ▨

7) $\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} = -\sec 2\theta$

$LS = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{1}{-\cos 2\theta}$

$= -\sec 2\theta$

$= RS$

Q.E.D ▨

8) $\frac{(\sin\theta + \cos\theta)^2}{\sin 2\theta} = \csc 2\theta + 1$

$LS = \frac{(\sin\theta + \cos\theta)^2}{\sin 2\theta}$

$= \frac{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}{\sin 2\theta}$

$= \frac{1 + \sin 2\theta}{\sin 2\theta}$

$= \frac{1}{\sin 2\theta} + \frac{\sin 2\theta}{\sin 2\theta}$

$= \csc 2\theta + 1$

$= RS$ Q.E.D ▨