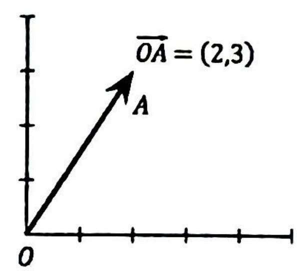
Day 6: 6.5 Vectors in R^2 and R^3

Points and Vectors in R^2:

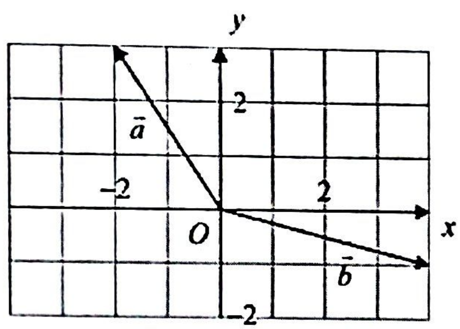
R^2 is a coordinate system constructed from two real number lines x and y which are perpendicular to each other and create a two-dimensional plane.
R^2 is made up of the xy-plane which expands infinitely in all directions.

A point $P(a, b)$ is determined by an ordered pair of real numbers a and b, and has a unique location in the coordinate system.

A vector can be drawn with its tail at $O(0,0)$ and its head at $P(a,b)$ and can be represented in **component form**, $\vec{OP} = (a,b)$, where a is called the x-component and b is called the y-component of $\vec{OP}$. Since $O(0,0)$ and $P(a,b)$ are unique points in the xy-plane, the associated vector, $\vec{OP}$, has a unique location in the xy-plane.



To locate $P(a,b)$, move a units from $O(0,0)$ along the x-axis and b units parallel to the y-axis. *Ex:* $\overrightarrow{OA} = (2, 3)$



Ex: Write the component form for vectors $\overrightarrow{OA}$ *and* $\overrightarrow{OB}$

$\vec{OA} = [-2, 3]$

$\vec{OB} = [4, -1]$

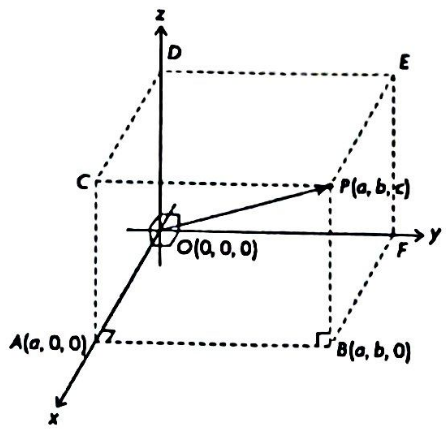
NOTE: I usually use [ ] for vectors and ( ) for points.

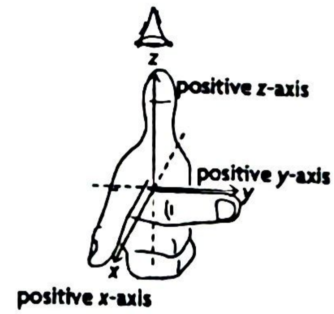
Points and Vectors in R^3:

R^3 is a coordinate system constructed from three real number lines x, y, and z which are all perpendicular to each other and create a three-dimensional system consisting of three two-dimensional planes, xy-plane, xz-plane, and yz-plane.

A point $P(a,b,c)$ is determined by real numbers a, b, c. $P(a,b,c)$ has a unique location in R^3 and its associated vector $\vec{OP} = (a,b,c)$ has a unique location.
Note: For $\vec{OP} = (a,b,c)$, a is the x-component, b is the y-component, and c is the z-component.

To locate points in $\mathbf{R}^3$ we use a right-handed system.





To locate $P(a,b,c)$, move a units from $O(0,0,0)$ along the x-axis, b units parallel to the y-axis, and c units parallel to the z-axis.

Constructing a rectangular box is often helpful in locating points in $\mathbf{R}^3$.

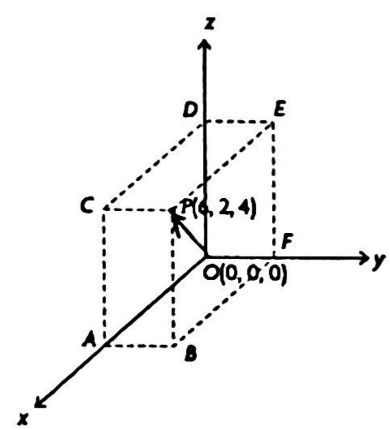
The position vector $\vec{OP}$ has its tail at $O(0,0,0)$ and its head at $P(a,b,c)$.

Points on the xy-plane are represented by $P(x,y,0)$ and the equation of the plane is $z = 0$.

Points on the xz-plane are represented by $P(x,0,z)$ and the equation of the plane is $y = 0$.

Points on the yz-plane are represented by $P(0,y,z)$ and the equation of the plane is $x = 0$.

Ex: a) In the following diagram, the point $P(6, 2, 4)$ is located in R^3. What are the coordinates of A, B, C, D, E, and F?
b) Draw the vector OP.



A (6,0,0)

B (6,2,0)

C (6,0,4)

D (0,0,4)

E (0,2,4)

F (0,2,0)

$\vec{OP}$ drawn in the box.

Ex: Given the point $P(4,-6,3)$

a) Determine the coordinates of the following points:

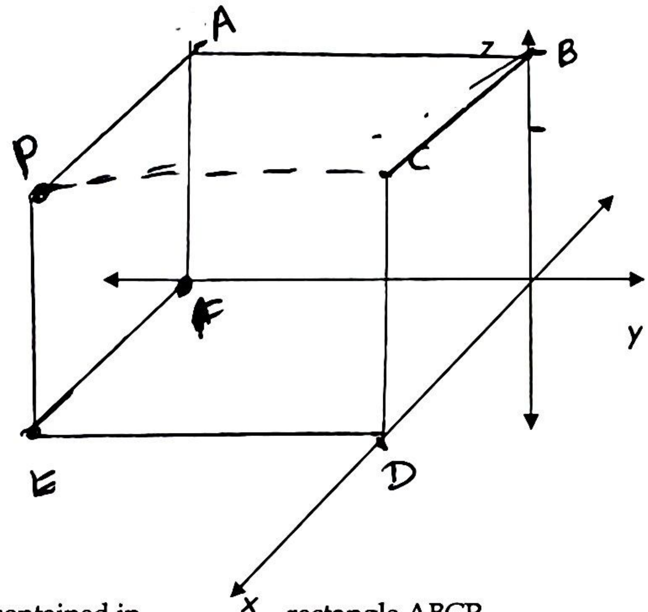
$A(0,-6,3)$	$B(0,0,3)$	$C(4,0,3)$
$D(4,0,0)$	$E(4,-6,0)$	$F(0,-6,0)$

b) What is the equation of the plane containing the points A, B, C, P?

NOTE: Points A, B, C, P have one thing in common. $Z=3$

$\therefore$ $z=3$ is the equation.

Recall: Equation of a horizontal line $y=3$ means y is 3 $x \in \mathbb{R}$.



c) Describe mathematically the set of points contained in rectangle ABCP.

All points in rectangle ABCP have the form $(x, y, 3)$ meaning x, y can be $\in \mathbb{R}$ but $z=3$.

$\therefore$ $\{x \in \mathbb{R} \mid 0 \le x \le 4\}$

$\{y \in \mathbb{R} \mid -6 \le y \le 0\}$