Day 6: 5.5 The Derivative of y = tan(x)

INVESTIGATION

Use quotient law to prove: y = tan(x)   y′ = sec²(x)

$$y = \frac{\sin x}{\cos x}$$

$$y' = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Example 1: Determine the derivative of the following

a) y = 3tan(2x)

$$y' = 3\sec^2(2x)\cdot 2$$

$$= 6\sec^2(2x)$$

b) y = tan²(x)

$$y = [\tan x]^2$$

$$y' = 2(\tan x)'(\sec^2 x)$$

$$\text{OR } y = \frac{2\sin x}{\cos^3 x}$$

c) y = sin[tan(x)]

$$y' = \cos[\tan x]\cdot(\tan x)'$$

$$= \cos[\tan x]\cdot\sec^2 x$$

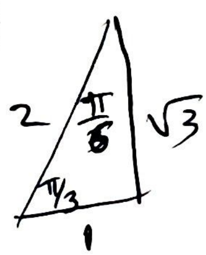
d) y = [tan(x² – x)]

$$y' = \sec^2(x^2 - x)\cdot(2x-1)$$

Example 2: Find the equation of the tangent lines to $y = \sin(x)\tan(\frac{x}{2})$ when $x = \frac{\pi}{3}$

$$y' = \cos x\tan\frac{x}{2} + (\sin x)(\sec^2(\frac{x}{2}))\cdot\frac{1}{2}$$

$$y'(\frac{\pi}{3}) = \cos\frac{\pi}{3}\tan(\frac{\pi}{6}) + \sin(\frac{\pi}{3})\sec^2(\frac{\pi}{6}))\cdot\frac{1}{2}$$



$$= (\frac{1}{2})(\frac{1}{\sqrt{3}}) + (\frac{\sqrt{3}}{2})(\frac{2}{\sqrt{3}})^2\cdot\frac{1}{2} = \frac{1}{2\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}} + \frac{2}{2\sqrt{3}}$$

$$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$x = \pi/3$
$y = 1/2$
$y' = m = \frac{\sqrt{3}}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2}(x - \pi/3) \Rightarrow y = \frac{\sqrt{3}}{2}x - \frac{\sqrt{3}\pi}{6} + \frac{1}{2}$$