Day 6: 2.4 -Families of Polynomial Functions
A family of polynomial functions refers to all polynomial functions that share some characteristic.

Polynomial functions are said to belong to the same family if they have:

- the same degree
- the same zeros of the same order


A family of polynomial functions may be represented in:
Factored General form:

$$
f(x)=k\left(x-x_{1}\right)\left(x-x_{2}\right) \ldots\left(x-x_{n}\right)
$$

Simplified General Form:

$$
f(x)=k\left(x^{n}+b_{n-1} x^{n-1}+\cdots+b_{1} x+b_{0}\right)
$$

Example One- The zeros for a cubic function are -4, 0, and 5
a) Write a general factored equation for the family of functions
b) Write the specific family member that goes through the point $(-1,14)$ in simplified general form.

$$
\left.\begin{array}{rl}
y=a(x+4)(x)(x-5) \\
x & =-1 \\
y=14
\end{array}\right\} \begin{aligned}
14 & =a(-1+4)(-1)(-1-5) \\
14 & =a(3)(6) \\
14 & =18 a \\
a & =\frac{14}{18} \\
a & =\frac{7}{9}
\end{aligned}
$$

Example Two - Determine the equation of the parabola in general simplified form that passes through the point $(5,6)$ and has roots of $\pm \sqrt{3}$.

$$
\begin{aligned}
& y=a(x+\sqrt{3})(x-\sqrt{3}) \\
& y=a\left(x^{2}-9\right) \\
& 6=a(25-9) \\
& 6=16 a \\
& a=\frac{3}{8} \\
& \therefore y=\frac{3}{8}\left(x^{2}-9\right)
\end{aligned}
$$

Example Three- A quartic function has zeros $\pm 2$ and $3 \pm \sqrt{2}$
a) Write a general simplified equation for the family of functions
b) Determine an equation of the member passing through $(1,-4)$
a)

$$
\begin{aligned}
y & =a(x+2)(x-2)(x-3-\sqrt{2})(x-3+\sqrt{2}) \\
& =a\left(x^{2}-4\right)\left((x-3)^{2}-2\right)=a\left(x^{2}-4\right)\left(x^{2}-6 x+7\right)
\end{aligned}
$$

b) Sub $x=1 \quad y=-4$

$$
\begin{aligned}
& -4=a(1-4)(1-6+7) \\
& 4=a(-3)(2) \\
& a=\frac{-4}{-6}=\frac{2}{3} \\
& \therefore y=\frac{2}{3}\left(x^{2}-4\right)\left(x^{2}-6 x+7\right)
\end{aligned}
$$

