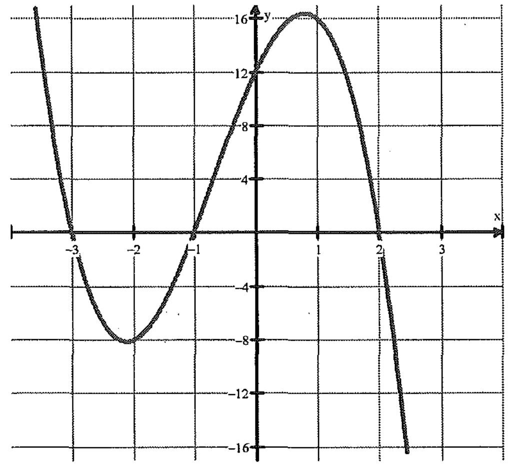
Day 6: 1.3-Equations and Graphs of Polynomial Functions

Example One: For the function below, determine the following:

a) The least possible degree and the sign of the leading coefficient

3. Sign should be negative since Q2 → Q4



b) The x-intercepts and the factors of the function

$y = -(x+3)(x+1)(x-2)$

since x-int are: -3, -1 and 2.

c) The intervals where the function is positive/negative

$x \in (-\infty, -3) \cup (-1, 2)$ positive

$x \in (-3, -1) \cup (2, \infty)$ negative.

> **Key Term:**
> ***Order:*** If a polynomial function has a factor $(x - a)$ that is repeated n times, then $x = a$ is a zero of order n.

Example Two: State the x – intercepts and their order of the following polynomial functions:

a) $y = (x-1)^2(x+2)$

$x = 1$ (order 2)

$x = -2$ (order 1)

b) $y = -x(x+4)^2(x-1)^3(2x-3)$

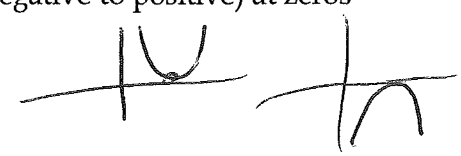
$x = -4$ (order 2)

$x = 0, \frac{3}{2}$ (order 1)

$x = 1$ (order 3)

Note: the graph of a polynomial function **changes sign** (positive to negative or negative to positive) at zeros that are of **odd order** and **does not change sign** at zeros that are of **even order.**

↓ bounces



We can use our previous knowledge of ***intercepts, order*** and ***end behavior*** to sketch the graph of a polynomial function.

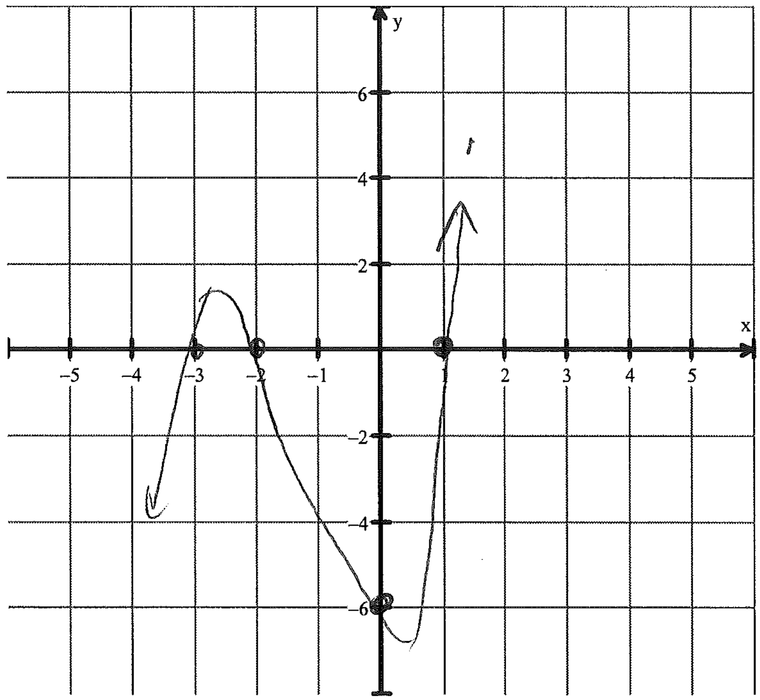
To graph a polynomial function:

Step one: Determine the degree and sign of the leading coefficient of the function
Step two: Use the degree and sign to determine the end behaviour
Step three: Factor and solve to find the x-intercepts (if not already factored)
Step four: Determine the order of the x-intercepts
Step five: Find the y-intercept
Step six: Use key points (intercepts) and end behavior to sketch

Example Three: Sketch a graph of each polynomial function.

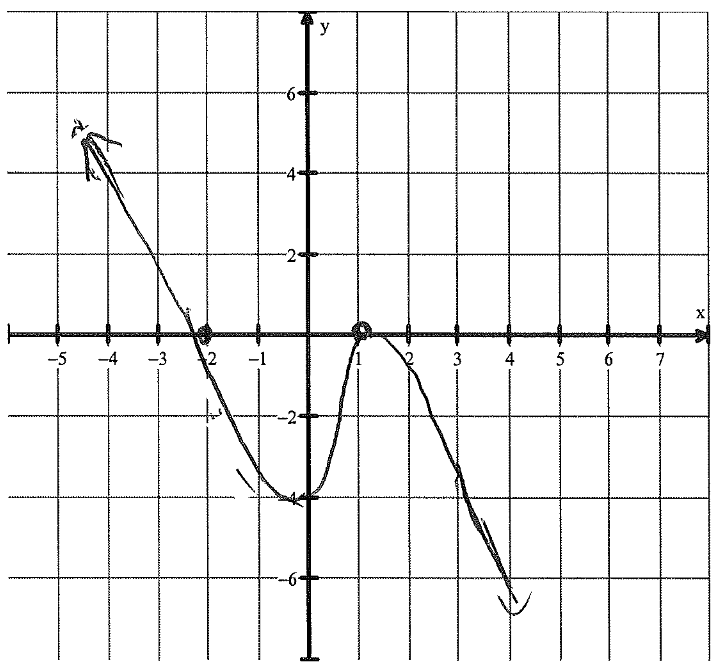
a) $y = (x-1)(x+2)(x+3)$

Degree	3
Leading Coefficient	+1
End Behaviour	Q1 → Q3
x – intercepts (& their order)	$x=1$ $x=-2$ (order 1) $x=-3$
y – intercept	$x=0$ $y=-6$



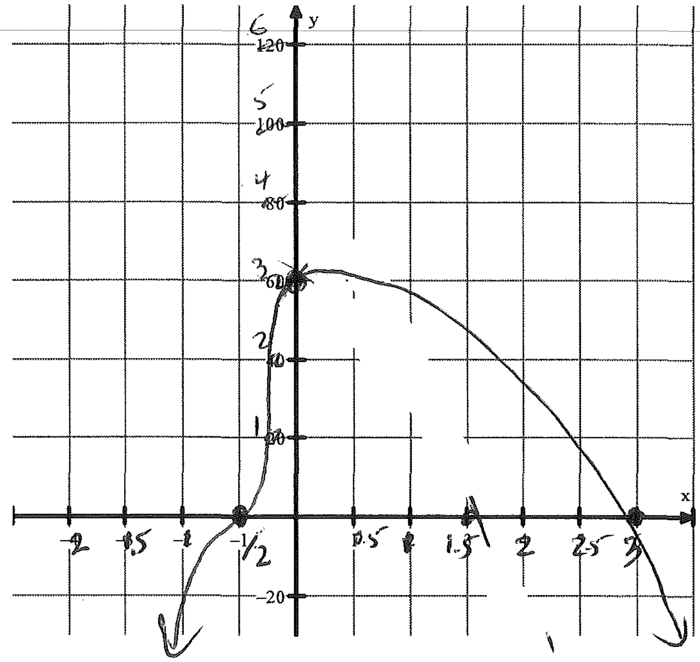
b) $y = -2(x-1)^2(x+2)$

Degree	3.
Leading Coefficient	−2
End Behaviour	Q2 → Q4
x – intercepts (& their order)	$x=1$ (order 2) $x=-2$ (order 1)
y – intercept	−4



c) $y = -(2x+1)^3(x-3)$

Degree	4
Leading Coefficient	-1
End Behaviour	Q3 → Q4
x – intercepts (& their order)	$x = -\frac{1}{2}$ (order 3) $x = 3$
y – intercept	3

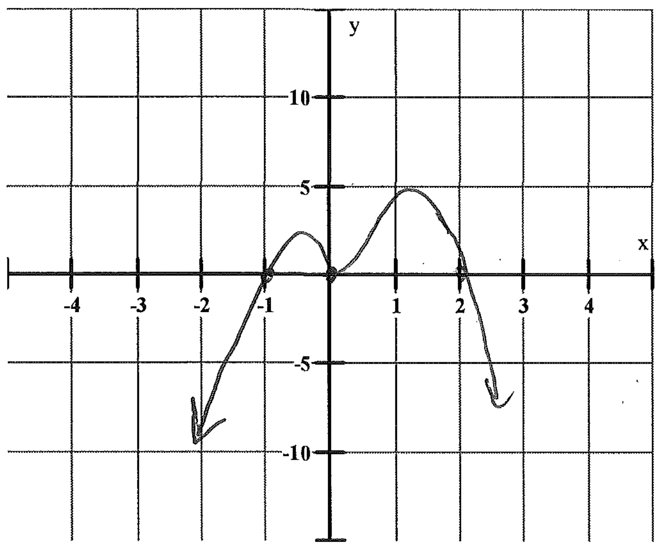


Example Four - Sketch the graph of a polynomial function, $f(x)$, that satisfies the following criteria:

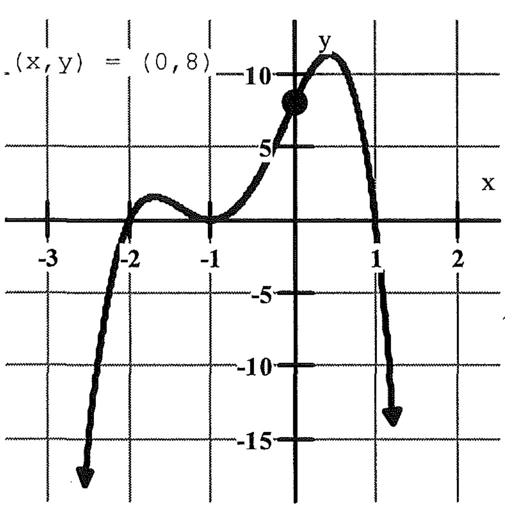
- Quartic function
- Negative leading coefficient
- Its only x-intercepts are at:

  $x = -1, x = 0, x = 2$

Since degree is 4, one of the zeros must be order 2.



Example Five - Determine the equation for a polynomial function given the following graph:



$y = a(x+2)(x+1)^2(x-1)$

sub $x=0$ $y=8$

$8 = a(0+2)(0+1)^2(0-1)$

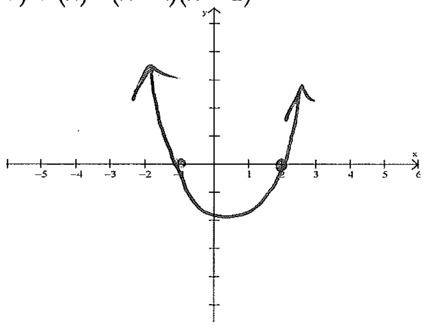
$-2a = 8 \Rightarrow a = -4$

$y = -4(x+2)(x+1)^2(x-1)$

What Role Do Factors Play?

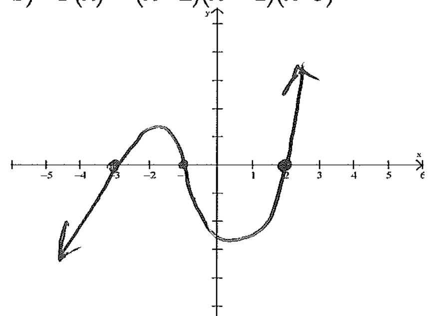
1. Sketch each polynomial function by identifying the x-intercepts, degree, and the end behaviour.

a) $f(x) = (x-2)(x+1)$



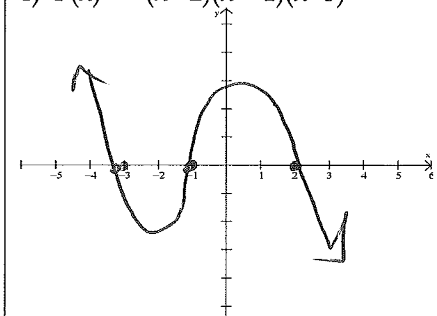
Degree of the function: 2 LC>0
x-intercepts: −1, 2

b) $f(x) = (x-2)(x+1)(x+3)$



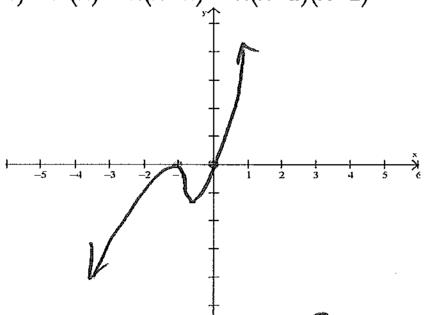
Degree of the function: 3 LC>0
x-intercepts: −3, −1, 2

c) $f(x) = -(x-2)(x+1)(x+3)$



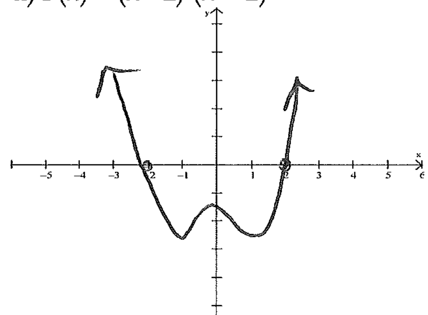
Degree of the function: 3 LC<0
x-intercepts: −3, −1, 2

c) $f(x) = x(x+1)^2 = x(x+1)(x+1)$



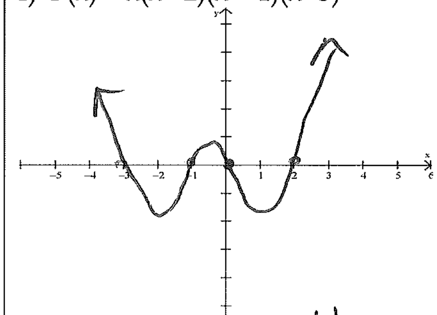
Degree of the function: 3
x-intercepts: −1 (order 2), 0

d) $f(x) = (x-2)^2(x+2)^2$



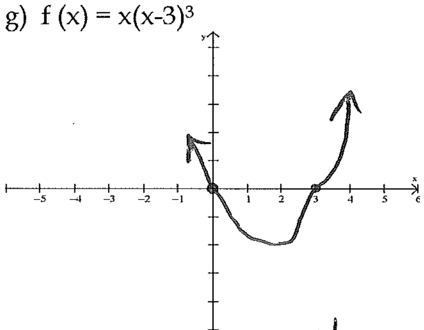
Degree of the function: 4
x-intercepts: x=2 (order 2), x=−2 (order 2)

f) $f(x) = x(x-2)(x+1)(x+3)$



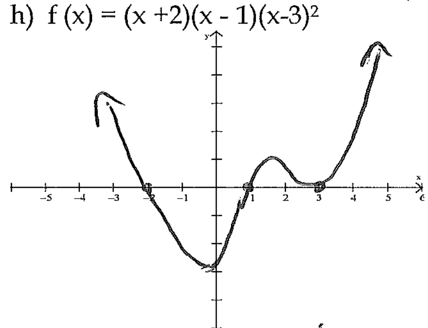
Degree of the function: 4
x-intercepts: −3, −1, 0, 2 (order 1)

g) $f(x) = x(x-3)^3$



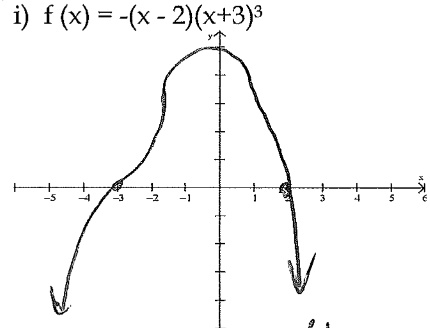
Degree of the function: 4
x-intercepts: 0, 3 (order 3)

h) $f(x) = (x+2)(x-1)(x-3)^2$



Degree of the function: 4
x-intercepts: −2, 1, 3 (order 2)

i) $f(x) = -(x-2)(x+3)^3$



Degree of the function: 4
x-intercepts: 2, −3 (order 3)

2. Compare your graphs with the graphs generated on the previous day and make a conclusion about the degree of a polynomial when it is given in factored form. Add the exponents. (provided linear in the bracket)

3. Explain how to determine the degree of a polynomial algebraically if given in factored form.

4. What connection do you observe between the factors of the polynomial function and the x-intercepts? Why does this make sense? (hint: all co-ordinates on the x axis have y = 0). $x-a$ has x-int of $x=a$
$x+b$ has x-int of $x=-b$

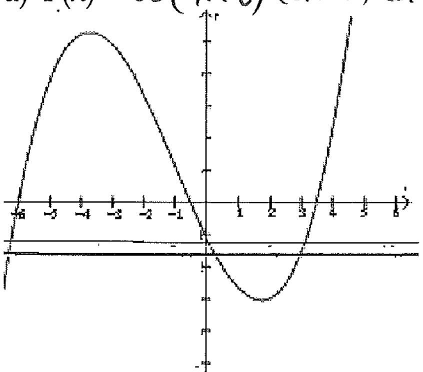
5. What do you notice about the graph when the polynomial function has a factor that occurs twice? Three times?
bounces. cubic

Source: Adapted from OAME MHF4U resources

What's My Polynomial Name?

1. Determine a possible equation for each polynomial function.

a) f (x)= $a(x+6)(2x-1)(2x-7)$



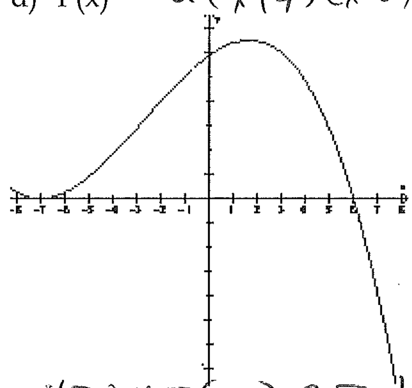
sub $x=0$ $y=-1$

$-1=a(6)(-1)(-7)$

$a=1/42$

$\therefore f(x)=1/42\,(x+6)(2x-1)(2x-7)$

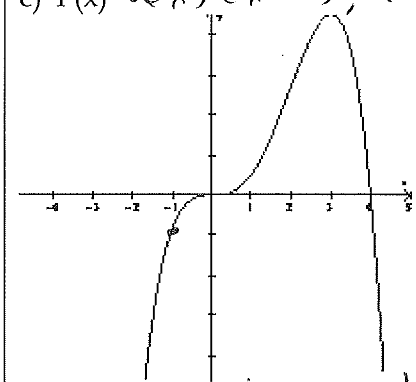
d) f (x) = $a(x+7)^2(x-6)$, $a<0$



$x=0$ $y=6 \Rightarrow a=-1/49$

$\therefore y=\frac{-1}{49}(x+7)^2(x-6)$

c) f (x) = $a(x)^3(x-4)$, $a<0$

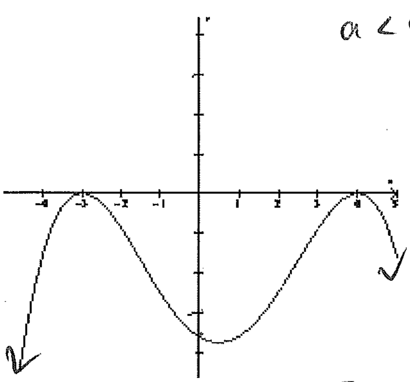


$x=-1$ $y=-1$ $a=\frac{-1}{5}$

$y=-\frac{1}{5}(x)^3(x-4)$

e) f (x) = $a(x+3)^2(x-4)^2$

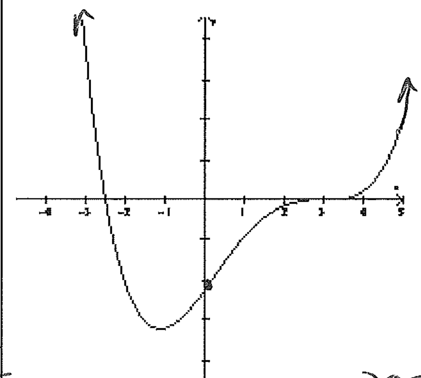
$a<0$



sub $x=0$ $y=-3.5$ $a=\frac{-7}{1440}$

$y=\frac{-7}{1440}(x+3)^2(x-4)^2$

d) f (x) = $a(2x+5)(x-3)^3$, $a>0$



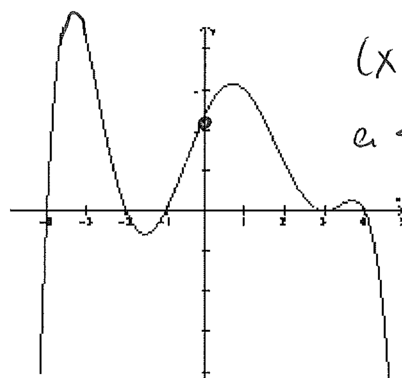
sub $x=0$ $y=-2 \Rightarrow a=\frac{2}{135}$

$y=\frac{2}{135}(2x+5)(x-3)^3$

f) f (x) = $a(x+3)(x+2)(x+1)(x-3)^2$.

$(x-4)$

$a<0$



$x=0$ $y=2 \Rightarrow a=1/-108$

$\therefore y=\frac{-1}{108}(x+3)(x+2)(x+1)(x-3)^2(x-4)$

2. Determine an example of an equation for a function with the following characteristics:

a) Degree 3, a double root at 4, a root at -3 $y=a(x-4)^2(x+3)$ $a\neq 0$

b) Degree 4, an inflection point at 2, a root at 5 $y=a(x-2)^3(x-5)$ $a\neq 0$

c) Degree 3, roots at ½, ¾, -1 $y=(2x-1)(4x-3)(x+1)$

d) Degree 3, starting in quadrant 2, ending in quadrant 4, root at -2 and double root at 3 $y=-(x+2)(x-3)^2$

e) Degree 4, starting in quadrant 3, ending in quadrant 4, double roots at -10 and 10 $y=-(x+10)^2(x-10)^2$

or $-(x^2-100)^2$

Source: Adapted from OAME MHF4U resources