

Day 6: 7.3 - Logarithm Laws

Warm Up - evaluate the following expressions using the logarithm laws.

$$\begin{aligned} \text{a) } \log_3 27^4 & \\ &= 4 \log_3 27 \\ &= 4(3) \\ &= 12 \end{aligned}$$

$$\begin{aligned} \text{b) } \log_5 250 - \log_5 2 & \\ &= \log_5 \left(\frac{250}{2} \right) \\ &= \log_5 125 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{c) } 2 \log 5 + \frac{1}{2} \log 16 & \\ &= \log 25 + \log 4 \\ &= \log 100 \\ &= 2 \end{aligned}$$

The product and quotient laws of logarithms, change of base formula, and the power law are useful tools for simplifying algebraic expressions and solving equations.

$$\log_b m = \frac{\log m}{\log b} \quad \log_b x^n = n \log_b x \quad \log_a x + \log_a y = \log_a xy$$

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

*Note: When we are working with variables to simplify logarithmic expressions, we need to state any restrictions. Remember, the domain of the logarithmic function is restricted.

EX 1 - Write as a sum or difference of logarithms. Simplify, if possible and *determine any restrictions*.

$$\begin{aligned} \text{a) } \log_3(xy) & \\ &= \log_3 x + \log_3 y \end{aligned}$$

$$\begin{aligned} \text{b) } \log \frac{uv}{\sqrt{w}} & \\ &= \log uv - \log \sqrt{w} \\ &= \log u + \log v - \log w^{1/2} \\ &= \log u + \log v - \frac{1}{2} \log w. \end{aligned}$$

EX 2 - Simplify the following algebraic expressions (Write as a single logarithm). Determine any restrictions.

a) $\log_3(x^2 - 1) - \log_3(x + 1)$

$$= \log_3 \frac{(x^2 - 1)}{(x + 1)}$$

$$= \log(x - 1), x > 1$$

$$x^2 - 1 > 0$$

$$x + 1 > 0$$



$$x > 1, x < -1$$

$$x > -1$$

∴ Overall, $x > 1$

b) $\log \sqrt[5]{x^3} + \log \sqrt{x} + \log \sqrt[5]{x}$

$$= \log x^{3/5} + \log x^{1/2} + \log x^{1/5}$$

$$= \log (x^{3/5} x^{1/2} x^{1/5})$$

$$= \log (x^{3/5 + 1/2 + 1/5})$$

$$= \log x \left(\frac{6}{10} + \frac{5}{10} + \frac{2}{10} \right)$$

$$= \log x^{13/10}, x > 0$$

c) $\log_4(x^2 - 2x - 3) - \log_4(x^2 + x - 2)$

$$= \log_4 \frac{(x^2 - 2x - 3)}{(x^2 + x - 2)}$$

$$= \log_4 \frac{(x - 3)(x + 1)}{(x - 2)(x + 1)}$$

$$= \log_4 \left(\frac{x - 3}{x - 2} \right), x < -1 \text{ or } x > 3$$

$$x^2 - 2x - 3 > 0$$



$$x^2 + x - 2 > 0$$

