Day 6 MPM 2D1

Triangle Centres

Date: _____

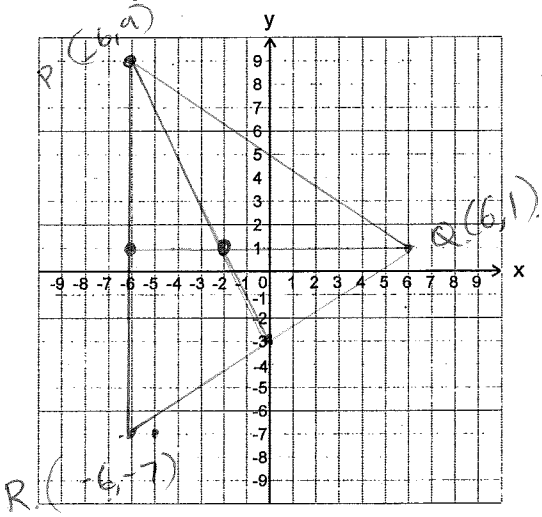
Use the glossary starting on page 570 in the textbook to define and illustrate the following:

1. The centroid of a triangle is POI of all the medians.

The centroid is also known as the Centre of the triangle

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Determine the centroid of the triangle with vertices P(-6,9) Q(6,1) and R(-6,-7) geometrically.



- we only find 2 equations and find POI.
- we can find the third one to check the answer

To determine the centroid algebraically:

median for vertex Q: $y=1$ (horizontal line)

median for vertex P: Midpoint RQ $= \left(\frac{-6+6}{2}, \frac{-7+1}{2}\right) = (0,-3)$

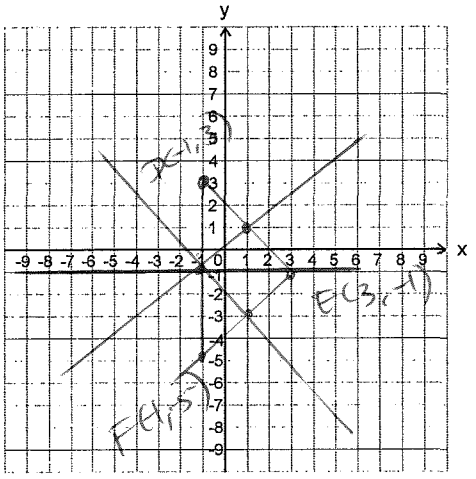
$m = \frac{-3-9}{0-(-6)} = \frac{-12}{6} = -2$ $\quad \therefore y = -2x - 3$

$\therefore$ POI is $(-2,1)$ since $y=1$

$1 = -2x - 3 \Rightarrow x = -2$

2. The circumcentre of a triangle is POI of all the right bisectors.

Determine the circumcentre of the triangle with vertices D(-1,3) E(3,-1) and F(-1,-5) geometrically.



FOR DE $M = \left(\frac{-1+3}{2}, \frac{3-1}{2}\right)$

$= (1, 1)$

$m_{DE} = \frac{-1-3}{3-(-1)} = \frac{-4}{4}$

$= -1$

$m_{\perp} = 1$ Use Midpoint.

$\therefore y = m(x-x_1)+y_1$

$= 1(x-1)+1$

$y = x$

To determine the circumcentre algebraically:

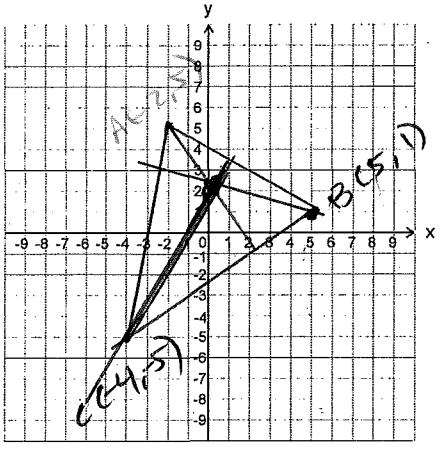
Right Bisector for DF: $y = -1$ (horizontal line).

" " for DE: $y = x$ [$m=1$, $b=0$]

$\therefore$ POI is $(-1,-1)$

3. The orthocentre of a triangle is POI of the altitudes.

Determine the orthocentre of the triangle with vertices A(-2,5), B(5,1) and C(-4, -5).



To determine the orthocentre algebraically:

Altitude for A(-2,5)

$m_{BC} = \frac{1+5}{5+4} = \frac{6}{9} = \frac{2}{3}$

$m_{\perp} = -\frac{3}{2}$ Point(-2,5)

$y = m(x-x_1)+y_1$

$= -\frac{3}{2}(x+2)+5$

$= -\frac{3}{2}x - 3 + 5$

$= -\frac{3}{2}x + 2$ (1)

Altitude for B.(5,1)

$m_{AC} = \frac{-5-5}{-4+2} = \frac{-10}{-2} = 5$

$m_{\perp} = -\frac{1}{5}$ point(5,1)

$y = m(x-x_1)+y_1$

$= -\frac{1}{5}(x-5)+1$

$= -\frac{1}{5}x + 1 + 1$

$= -\frac{1}{5}x + 2$ (2)

POI of (1) & (2)

$y = \frac{-3}{2}x + 2$ $\quad y = \frac{-1}{5}x + 2$

Since y-int is 2

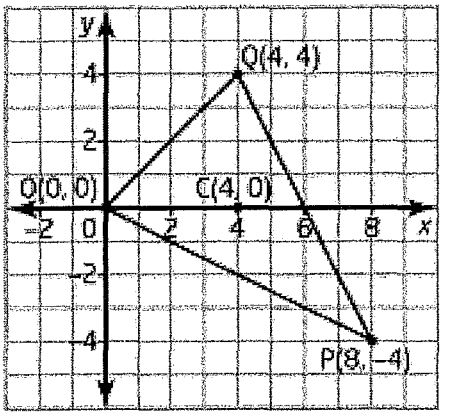
POI is (0, 2)

For any triangle the orthocentre, circumcentre and centroid are Collinear (on the same line)

Properties of Triangles

Ex1. a) Verify that C(4,0) is the centroid of OPQ triangle.

b) Verify that the centroid divides each median in a 2:1 ratio.



Ex2. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length. Verify algebraically this property in triangle A(−10,1), R (6,5) and T(2, −11). Include a labelled diagram as part of your solution.