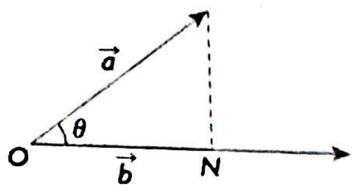
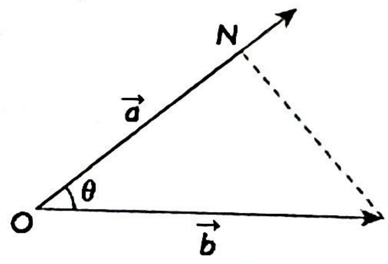
Day 5: 7.5 Scalar and Vector Projections

<u>Projections</u> are formed by dropping a perpendicular from the head of one vector to another vector, or an extension of another vector. (Can be thought of as a shadow)

Given two vectors $\vec{a}$ *and* $\vec{b}$, think of the projection of $\vec{a}$ *on* $\vec{b}$ as the shadow that $\vec{a}$ casts on $\vec{b}$

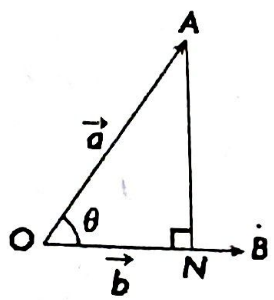


Projection of $\vec{a}$ onto $\vec{b}$

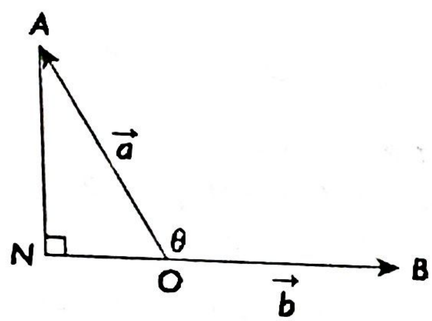


Projection of $\vec{b}$ onto $\vec{a}$

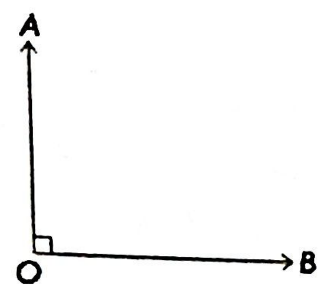
The direction of the projection of $\vec{a}$ *on* $\vec{b}$ depends on the angle θ between $\vec{a}$ *and* $\vec{b}$



$0 \leq \theta < 90°$



$90° < \theta \leq 180°$



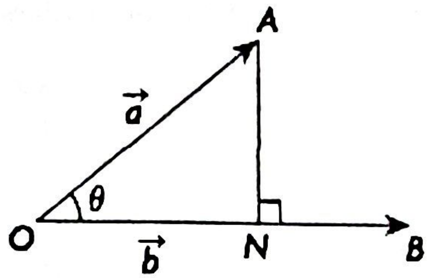
$\theta = 90°$

Calculating Scalar Projections

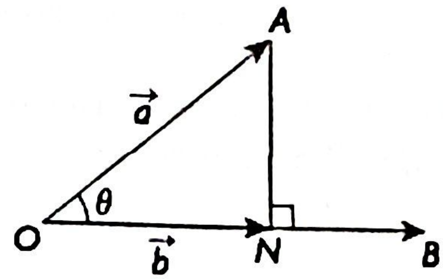
The scalar projection of $\vec{a}$ on $\vec{b}$ is $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

The scalar projection of $\vec{b}$ on $\vec{a}$ is $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$.

In general, $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \neq \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$.



Scalar projection of $\vec{a}$ on $\vec{b}$



Vector projection of $\vec{a}$ on $\vec{b}$

Vector Projection of $\vec{a}$ on $\vec{b}$

vector projection of $\vec{a}$ on $\vec{b}$

= (scalar projection of $\vec{a}$ on $\vec{b}$) (unit vector in the direction of $\vec{b}$)

$$= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\left(\frac{\vec{b}}{|\vec{b}|}\right)$$

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$$

$$= \left(\frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\right)\vec{b},\ \vec{b} \neq \vec{0}$$

Example1: Find the scalar and vector projections of $\vec{u} = (1, 3, 5)$ *on* $\vec{v} = (-1, 3, -2)$.

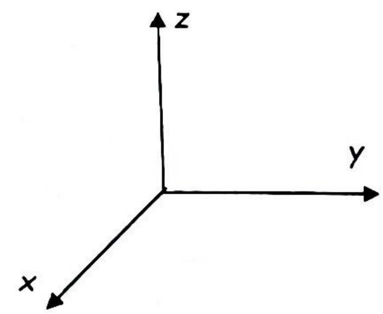
Scalar projection of $\vec{u}$ on $\vec{v}$ $= \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$

$$= \frac{(1)(-1) + 3(3) + 5(-2)}{\sqrt{(-1)^2 + (3)^2 + (-2)^2}}$$

$$= \frac{-2}{\sqrt{14}}$$

The angles that a vector $\vec{OP}$ makes with each positive axis are called **direction cosines**.

α is the angle $\vec{OP}$ makes with the positive x-axis.
β is the angle $\vec{OP}$ makes with the positive y-axis.
γ is the angle $\vec{OP}$ makes with the positive z-axis.



Direction Cosines for $\overrightarrow{OP} = (a, b, c)$

If α, β, and γ are the angles that $\overrightarrow{OP}$ makes with the positive x-axis, y-axis, and z-axis, respectively, then

$$\cos\alpha = \frac{(a,b,c)\cdot(1,0,0)}{|\overrightarrow{OP}|} = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$\cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}} \text{ and } \cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Example2: Determine the direction angles for $\vec{u} = (1, 3, 5)$.

$$\cos\alpha = \frac{(1,3,5)\cdot(1,0,0)}{\sqrt{1^2+3^2+5^2}}$$

$$\cos\alpha = \frac{1}{\sqrt{35}} \qquad \cos\beta = \frac{3}{\sqrt{35}} \qquad \cos\gamma = \frac{5}{\sqrt{35}}$$

$$\alpha \doteq 80^\circ \qquad \beta \doteq 60^\circ \qquad \gamma \doteq 32^\circ$$