Day 5: 6.4 Properties of Vectors

Addition and Scalar Multiplication	
1. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$	2. $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$
3. $\vec{a}+\vec{0}=\vec{a}$	4. $\vec{a}+(-\vec{a})=\vec{0}$
5. $c(\vec{a}+\vec{b})=c\vec{a}+c\vec{b}$	6. $(c+d)\vec{a}=c\vec{a}+d\vec{a}$
7. $(cd)\vec{a}=c(d\vec{a})$	8. $1\vec{a}=\vec{a}$

Ex 1: Simplify the following expression.

$$4(\vec{a}-2\vec{b}+\vec{c})-(5\vec{a}+3\vec{b}-2\vec{c})-2(-\vec{a}+\vec{b}-3\vec{c})$$

$= 4\vec{a}-8\vec{b}+4\vec{c}-5\vec{a}-3\vec{b}+2\vec{c}+2\vec{a}-2\vec{b}+6\vec{c}$

$= 4\vec{a}-5\vec{a}+2\vec{a}-8\vec{b}-3\vec{b}-2\vec{b}+4\vec{c}+2\vec{c}+6\vec{c}$

$= \vec{a}-13\vec{b}+12\vec{c}$

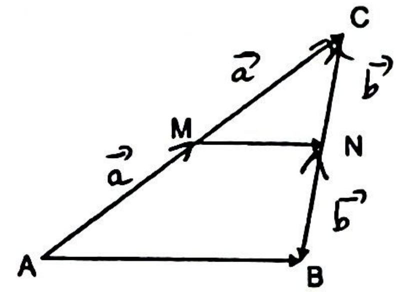
Ex 2: If $\vec{x}=2\vec{i}-3\vec{j}+\vec{k}$ and $\vec{y}=\vec{i}-3\vec{k}$, determine $2\vec{x}-\vec{y}$ in terms of $\vec{i}$, $\vec{j}$, $\vec{k}$.

$2\vec{x}-\vec{y} = 2(2\vec{i}-3\vec{j}+\vec{k})-(\vec{i}-3\vec{k})$

$= 4\vec{i}-6\vec{j}+2\vec{k}-\vec{i}+3\vec{k}$

$= 3\vec{i}-6\vec{j}+5\vec{k}$

Ex 3: Consider the triangle ΔABC. Let M be the midpoint of AC and N be the midpoint of BC.
Prove that $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AB}$

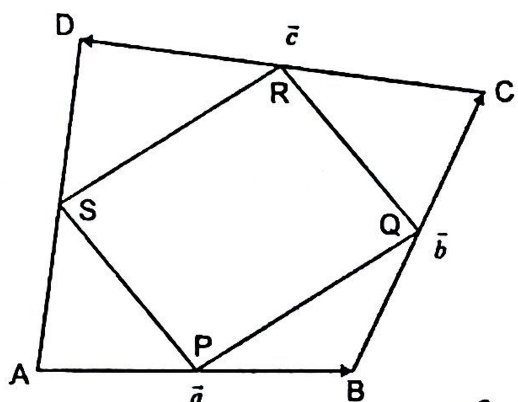


$$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN}$$
$$= \vec{a} - \vec{b}$$

$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$$
$$= 2\vec{a} - 2\vec{b}$$
$$= 2(\vec{a} - \vec{b}) = 2\overrightarrow{MN}$$

$$\therefore \overrightarrow{AB} = 2\overrightarrow{MN}$$
$$\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AB}$$

Ex 4: ABCD is a quadrilateral where *P, Q, R,* and *S* are the midpoints of *AB, BC, CD, and DA* respectively. Prove, using vector methods, that *PQRS* is a parallelogram.



$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$$
$$\overrightarrow{AD} = \vec{a} + \vec{b} + \vec{c}$$

$$PQ = \overrightarrow{PB} + \overrightarrow{BQ}$$
$$= \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b}$$

$$SR = \overrightarrow{SD} + \overrightarrow{DR}$$
$$= \frac{1}{2}\overrightarrow{AD} - \frac{1}{2}\vec{c}$$
$$= \frac{1}{2}(\vec{a} + \vec{b} + \vec{c}) - \frac{1}{2}\vec{c}$$
$$= \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} \quad \therefore \overrightarrow{PQ} \parallel \overrightarrow{SR}$$

parallel

$$\overrightarrow{SP} = \overrightarrow{SA} + \overrightarrow{AP}$$
$$= \frac{1}{2}(\overrightarrow{AD}) + (+\frac{1}{2}\vec{a})$$
$$= -\frac{1}{2}(\vec{a} + \vec{b} + \vec{c}) + \frac{1}{2}\vec{a}$$
$$= -\frac{1}{2}\vec{b} - \frac{1}{2}\vec{c}$$

$$\cancel{-\frac{1}{2}\vec{a}} - \frac{1}{2}\vec{b} - \frac{1}{2}\vec{c} + \cancel{\frac{1}{2}\vec{a}}$$

$$\overrightarrow{RQ} = \overrightarrow{RC} + \overrightarrow{CQ}$$
$$= -\frac{1}{2}\vec{c} - \frac{1}{2}\vec{b}$$

$$\overrightarrow{SP} \parallel \overrightarrow{RQ}$$

$\therefore$ PQRS is a parallelogram.