2.3 –*Solving Polynomial Equations*

To **solve a polynomial equation**, we must determine any values of x that make the equation true.

To do this, write the equation in **factored form**, then set each **factor equal to zero**, and **solve for x**

Example One– Solve the following polynomials by factoring

a) $x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$x(x-2)(x+1) = 0$

$x = \{-1, 0, 2\}$

b) $3x^3 + x^2 - 12x - 4 = 0$ → factor by grouping.

$x^2(3x+1) - 4(3x+1) = 0$

$(x^2-4)(3x+1) = 0$

$(x-2)(x+2)(3x+1) = 0$

$\therefore x = \{-2, -\frac{1}{3}, 2\}$

Example Two – Solve the following polynomial by using the factor theorem

$2x^3 + 3x^2 - 11x - 6 = 0$ $\frac{b}{a}$: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

let $P(x) = 2x^3 + 3x^2 - 11x - 6$

$P(1) \neq 0$ $P(-1) = -2 + 3 + 11 - 6 \neq 0$ $P(2) = 16 + 12 - 22 - 6 = 0$

$\Rightarrow x - 2$ is a factor

$$\begin{array}{c|cccc} 2 & 2 & 3 & -11 & -6 \\ & \downarrow & 4 & 14 & 6 \\ \hline & 2 & 7 & 3 & 0 \end{array}$$

$\therefore (x-2)(2x^2 + 7x + 3) = 0$

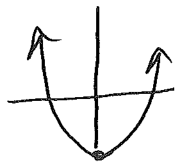
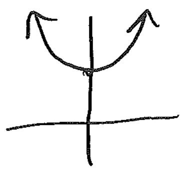
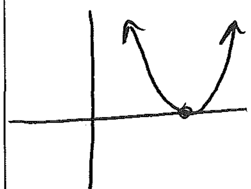
$(x-2)(2x+1)(x+3) = 0$

$x = \{-3, -\frac{1}{2}, 2\}$

NOTE: If a quadratic does not factor, use quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

How are roots, x-intercepts, and zeros related? The key distinction is context:

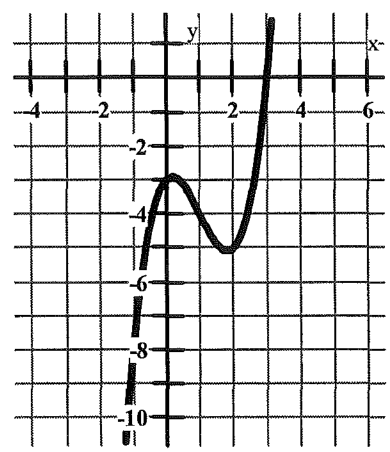
Equation	Sketch of Graph	Roots (of an equation)	Zeros (of a function)	x-intercepts (of a graph)
$y = x^2 - 4$		$x = \{\pm 2\}$	$x = 2$ or $x = -2$	$(2,0)$ or $(-2,0)$
$y = x^2 + 4$		No Real roots. complex roots: $x = \{\pm 2i\}$	NO REAL ZEROS (COMPLEX ZEROS EXIST) $x=2i$ or $x=-2i$	NONE
$y = (x-4)^2$		$x = \{4\}$ order 2	$x = 4$ order 2	$(4,0)$ order 2

* If a polynomial equation is factorable, the roots are determined by factoring the polynomial, setting its factors equal to zero, and solving for each factor

* If a polynomial equation is not factorable, the roots can be determined from the graph or the quadratic formula

A polynomial equation may have real and non-real roots

Example Three- Consider the solution to the polynomial equation
$x^3 - 3x^2 + x - 3 = 0$



* The x-intercepts of the graph correspond to the real roots of the related polynomial equation

$x = 3$ is the real solution
There are 2 complex roots.
} hence it is a cubic