Day 5: 1.2-Characteristics of Polynomial Functions

Continued

Key Terms:
Intervals of Increase - interval(s) where y increases as x increases
Intervals of Decrease - interval(s) where y decreases as x increases

Positive Intervals – interval(s) where the function lies above the x-axis
Negative Intervals – interval(s) where the function lies below the x-axis

Example One: Answer the questions based on the function below:

a) State the intervals of increase and decrease (using **interval notation**)

$x \in (-\infty, 0) \cup (1.6, \infty)$ INCREASE

$x \in (0, 1.6)$ DECREASE

b) State the positive and negative intervals (using **interval notation**)

$x \in (-1, 1) \cup (2, \infty)$ $f(x) > 0$

$x \in (-\infty, -1) \cup (1, 2)$ $f(x) < 0$

c) State the least possible degree of the function.

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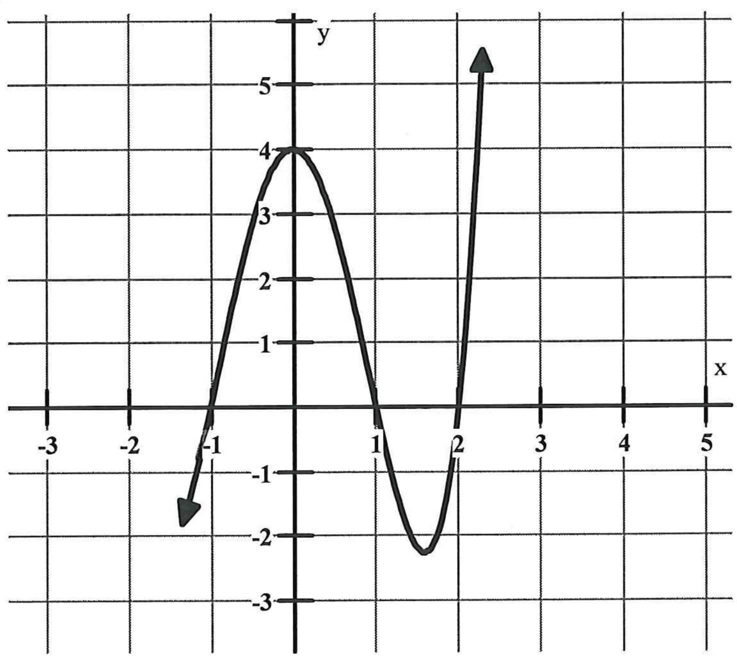

d) State the sign of the leading coefficient of the function.

Positive since Q1→Q3

Finite Differences

For a polynomial function of degree n, where n is a positive integer, the n-th differences:

- Are equal (or constant)
- Have the same sign as the leading coefficient
- Are equal to $a[(n)(n-1)(n-2)\ldots(2)(1)]$, where a is the leading coefficient. In other words, $an! = n^{th}$ differences. eg. for quadratic: $2a = 2^{nd}$ differences.

Note: 5! is read as 5 factorial and is equal to 5x4x3x2x1=120

cubic $6a = 3^{rd}$ differences.
quartic $24a = 4^{th}$ "

Example Two: Use finite differences to determine the following:

x	Y	1st Differences	2nd Differences	3rd Differences	4th Differences
-2	-54				
		-8-(-54)=46			
-1	-8		8-46=-38		
		0-(-8)=8		-2+38=36	
0	0		6-8=-2		12-36=-24
		6-0=6		10+2=12	
1	6		16-6=10		-12-12=-24
		22-6=16		-2-10=-12	
2	22		14-16=-2		-36+12=-24
		36-22=14		-38+2=-36	
3	36		-24-14=-38		-68+36=24
		12-36=-24		-98+38=-68	
4	12		-122+24=98		
		-110-12=-122			
5	-110				

a) degree of polynomial

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b) sign of leading coefficient

negative since 4th differences are negative.

c) value of the leading coefficient:

$a(4\times3\times2\times1) = -24$

$24a = -24$

$\boxed{a = -1}$

Example Three: For the following polynomial, determine which finite difference is constant and its value

$$f(x) = -2x^4 + 8x$$

4th diferences would be constant

$a(4\times3\times2\times1) =$ 4th differences

$24(-2) = -48 =$ 4th differences (value).