Day 5: 6.4 – Logarithm Laws

Investigation:

1) Evaluate the following:

a) $\log 1$

$10^{?} = 1$

$\log 1 = 0$

b) $\log 10$

$\log 10 = 1$

c) $\log 100$

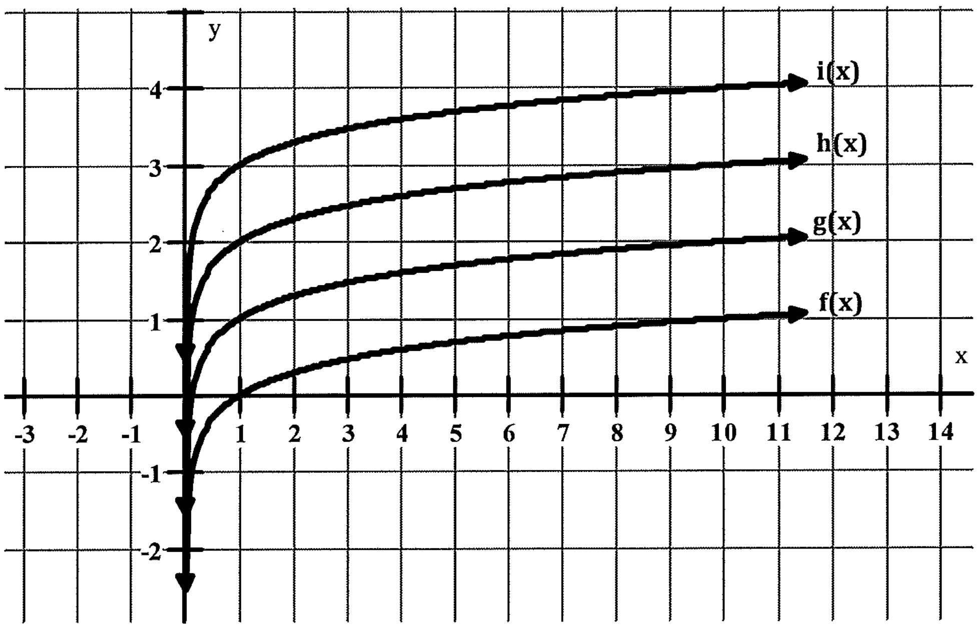
$\log 100 = 2$

d) $\log 1000$

$\log 1000 = 3$

2) The following graphs are plotted on the same grid:

$f(x) = \log x$ $\quad$ $g(x) = \log(10x)$ $\quad$ $h(x) = \log(100x)$ $\quad$ $i(x) = \log(1000\,x)$



Describe how these graphs are related to each other using vertical translations.

$g(x)$: 1 unit up.

$h(x)$: 2 units up.

$i(x)$: 3 units up.

3) Fill in the chart below, writing each function in 3 different ways.

Function	Vertical Translation of $f(x)$	Sum of Common Logarithms
$g(x) = \log(10x)$	1 unit up.	$\log 10x = \log(10) + \log x$ $= 1 + \log x$
$h(x) = \log(100x)$	2 units up	$\log 100x = \log 100 + \log x$ $= 2 + \log x$
$i(x) = \log(1000\,x)$	3 units up.	$i(x) = \log x + 3$

What can we conclude from this investigation?

$\log(ab) = \log a + \log b$

Product Law of Logarithms:

$$\log_a x + \log_a y = \log_a xy$$

Quotient Law of Logarithms:

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

$a > 0, a \neq 1, x > 0, y > 0$

EX 1 – Simplify the following to **write as a single logarithm.**

a) $\log 11 + \log 4$

$= \log[(11)(4)]$

$= \log 44$

b) $\log_5 6 + \log_5 8 - \log_5 16$

$= \log_5 \left[\frac{6 \cdot 8}{16}\right]$

$= \log_5 \left[\frac{48}{16}\right]$

$= \log_5 3$

EX 2 – Evaluate the following.

a) $\log_2 4 + \log_2 8$

$= \log_2[(4)(8)]$

$= \log_2 32$

$= 5$

b) $\log 20^2 - \log\sqrt{16}$

$= \log \frac{20^2}{\sqrt{16}}$

$= \log\left(\frac{400}{4}\right)$

$= \log 100$

$= 2$

Power Law of Logarithms:

$$\log_b x^n = n\log_b x$$

$b > 0, b \neq 1, x > 0, n \in R$

Proof 1 $LS = \log_b x^n = \log_b (\underbrace{x \cdot x \cdot x \cdot x \ldots x}_{n \text{ times}})$

$= \underbrace{\log_b x + \log_b x + \ldots + \log_b x}_{n} = n\log_b x = RS$

QED.

Proof 2: Let $P = \log_b x^n$ $Q = n\log_b x$ we need to show $P = Q$

$b^P = x^n$ $b^{\frac{Q}{n}} = x$

$\therefore b^P = (b^{\frac{Q}{n}})^n \Rightarrow b^P = b^Q \Rightarrow P = Q$

Q.E.D

EX 3 - Using the power law of logarithms – evaluate the following:

a) $\log_3 9^4$

$= 4\log_3 9$

$= 4(2)$

$= 8$

b) $\log_2 8^5$

$= 5\log_2 8$

$= (5)(3)$

$= 15$

c) $\log 0.001^2$

$= 2\log 0.001$

$= 2(-3)$

$= -6$

d) $\log_5 \sqrt{125}$

$= \log_5 (125)^{1/2}$

$= \frac{1}{2}\log_5 125$

$= \frac{1}{2}(3)$

$= 3/2$

e) $\log_3 12 + \frac{1}{2}\log_3 36 - \log_3 8$ ***Need to use product, quotient, and power law**

$= \log_3 (12) + \log_3 \sqrt{36} - \log_3 8$

$= \log_3 (12) + \log_3 6 - \log_3 8$

$= \log_3 \left(\frac{(12)(6)}{8}\right) = \log_3 9 = 2$

Change of Base Formula:

$$\underset{P}{\log_b m} = \frac{\log m}{\log b} \quad \begin{matrix} = Q \\ = R \end{matrix}$$

$m > 0, b > 0, b \neq 1$

Proof: Let $P = \log_b m$ $\quad Q = \log m \quad R = \log b$

we need to show $P = \frac{Q}{R}$

$b^P = m \qquad 10^Q = m \qquad 10^R = b.$

$b^P = 10^Q$ $\quad$ sub $b = 10^R$

$(10^R)^P = 10^Q$

$10^{PR} = 10^Q \Rightarrow PR = Q$

$P = \frac{Q}{R}$

EX 4 - Using the change of base formula – Evaluate the following:

**** You would need your calculator to get an approximate answer***

a) $\log_5 17$

$= \frac{\log 17}{\log 5}$

$\doteq 1.76$

b) $\log_{\frac{1}{2}} 10$

$= \frac{\log 10}{\log 1/2}$

$= -3.32$