Day 4: 7.4 The Dot Product of Algebraic Vectors

Recall: Algebraic vectors in component form $\vec{a} = (a_1, a_2, a_3)$ in R³.

In R^2, if $\vec{a} = (a_1, a_2)$ *and* $\vec{b} = (b_1, b_2)$ *then* $\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a} = a_1b_1 + a_2b_2$.

In R^3, if $\vec{a} = (a_1, a_2, a_3)$ *and* $\vec{b} = (b_1, b_2, b_3)$ *then* $\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a} = a_1b_1 + a_2b_2 + a_3b_3$.

Ex 1: Given $\vec{u} = (3, 2, -1)$ *and* $\vec{v} = (-1, 4, 5)$, find $\vec{u} \bullet \vec{v}$.

$\vec{u} \cdot \vec{v} = (3)(-1) + 2(4) + (-1)(5)$

$= -3 + 8 - 5$

$= 0$ [since the dot product is 0, they are perpendicular] as $\cos 90° = 0$.

Using Dot product to find the angle between 2 vectors

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}:$$

if $\cos\theta = 0 \Leftrightarrow \theta = 90°$

$\cos\theta > 0 \Leftrightarrow 0° < \theta < 90°$

$\cos\theta < 0 \Leftrightarrow 90° < \theta < 180°$

Ex2: Determine the angle between $\vec{a} = (3, 4)$ and $\vec{b} = (-2, -3)$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{(3)(-2) + (4)(-3)}{\sqrt{3^2 + 4^2}\sqrt{(-2)^3 + (-3)^2}}$$

$$\cos\theta = \frac{-18}{(5)\sqrt{13}} \Rightarrow \theta = 176.8°$$

Ex3: Determine the value of k so that $\vec{u} = (2, 5)$ *and* $v(k, 4)$ are perpendicular (**orthogonal**).

perpendicular $\Rightarrow$ dot product $= 0$

$(2)(k) + (5)(4) = 0$

$2k = -20$

$\boxed{k = -10}$

Ex4: A parallelogram has its sides determined by $\vec{a} = (2, 3)$ and $\vec{b} = (3, 1)$. Determine the angle between the diagonals of the parallelogram formed by these vectors.

$\vec{a}+\vec{b} = [5, 4]$

$a-\vec{b} = [-1, 2]$

$$\cos\theta = \frac{(5)(-1) + (4)(2)}{\sqrt{5^2+4^2}\ \sqrt{(-1)^2+(2)^2}}$$

$$\cos\theta = \frac{3}{\sqrt{41}\sqrt{5}}$$

$$\theta \doteq 77.9^\circ$$

Ex5: Given vectors $\vec{a} = (2, -1, 3)$ and $\vec{b} = (1, -4, 2)$, determine the components of a vector **perpendicular to each** of these vectors.

Let $\vec{c} = (x, y, z)$

$\vec{a}\cdot\vec{c} = 0 \Rightarrow 2x - y + 3z = 0$ ①

$\vec{b}\cdot\vec{c} = 0 \Rightarrow x - 4y + 2z = 0$ ②

we need 3 equations to solve 3 variables. So, we can freeze one variable then find the conditions.

① : $2x - 1y + 13z = 0$

②×2 : $2x - 8y + 4z = 0$

$7y - z = 0$

$z = 7y$

①×2 : $4x - 2y + 6z = 0$

②×3 : $3x - 12y + 6z = 0$

$x + 10y = 0$

$x = -10y$

∴ Given y,

$x = -10y$ and $z = 7y$

eg. $(-10, 1, 7)$ OR $(-20, 2, 14)$

Ex6: Suppose that a force vector given by $\vec{F} = (-2, -6, 3)$ moves an object from point **A**(3, -1, 2) to point **B**(1, 4, 4). Calculate the <u>work</u> done on the object.

$W = |\vec{F}\cdot\vec{s}|$

absolute value

$\vec{s}$ is the displacement vector

$\vec{s} = \overrightarrow{AB} = (1-3, 4-(-1), 4-2)$

$= (-2, 5, 2)$

$\vec{F}\cdot\vec{s} = (-2, -6, 3)\cdot(-2, 5, 2)$

$= 4 - 30 + 6$

$= -20$

∴ Work done is 20 Joules.