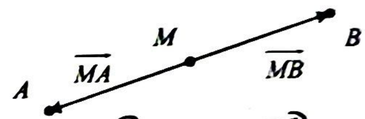
Day 4: 6.3 Multiplication of a Vector by a Scalar

Scalar Multiplication:
From the previous lesson, we notice that two vectors are parallel, one vector can be expressed in terms of the other.

Ex: Midpoint
The midpoint of the segment line AB is the point M such that $\overrightarrow{MA} + \overrightarrow{MB} = \vec{0}$



What else can we say? $|\overrightarrow{MA}| = |\overrightarrow{MB}|$, $|\overrightarrow{AB}| = 2|\overrightarrow{MB}|$, $\overrightarrow{AB} = 2\overrightarrow{MA}$.

$\vec{a}$ is a nonzero vector, multiplied by a scalar k, we obtain a new vector $k\vec{a}$, such that: IF $\vec{a}$ →

$k > 1$, vector gets longer and stays in the same direction. $2\vec{a}$ →

$0 < k < 1$, vector gets shorter and stays in the same direction. → $0.5\vec{a}$

$-1 < k < 0$, vector gets shorter and goes in the opposite direction. ← $-0.5\vec{a}$

$k < -1$, vector gets longer and goes in the opposite direction. ← $-2\vec{a}$.

$k = 0$, result is the zero vector. $\vec{0}$ has every direction.

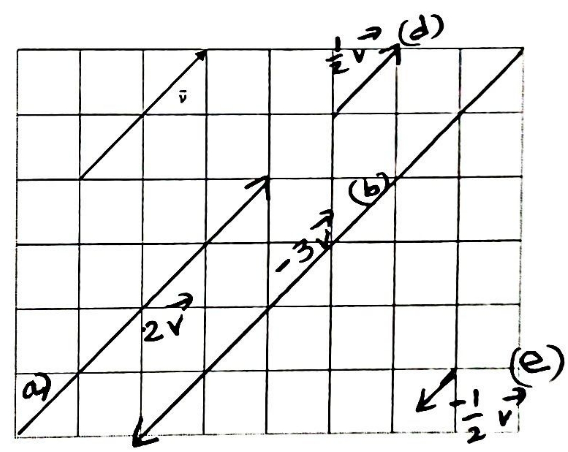
$k = -1$, result is vector of same length but opposite direction. ← $-\vec{a}$

Example: compare $-\frac{2}{3}\vec{a}$ with $\vec{a}$ $-\frac{2}{3}\vec{a}$ would be shorter and in opposite direct.

The magnitude of $\overrightarrow{ka}$ is $|\overrightarrow{ka}| = |k| \times |\vec{a}|$

Given the vector $\vec{v}$ draw the following vectors

a) $2\vec{v}$ b) $-3\vec{v}$ d) $\frac{1}{2}\vec{v}$ e) $-\frac{1}{4}\vec{v}$



Properties of Scalar Multiplication, m, n, are scalars, $\vec{a}$, $\vec{b}$, are nonzero vectors:

$(mn)\vec{a} = m(n\vec{a})$ (Associative Law)
$m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$ (Distributive Law)
$(m+n)\vec{a} = m\vec{a} + n\vec{a}$

Two vectors that lie on the same line or are parallel and can be translated to lie on the same line are said to be collinear. (Note: When describing vectors, parallel and collinear can be use interchangeably.)

Collinear Vectors

Two vectors $\vec{u}$ and $\vec{v}$ are collinear iff it is possible to find a nonzero scalar k such that $\vec{u} = k\vec{v}$.

Unit Vectors are vectors with **magnitude 1**. A unit vector in the direction of any vector $\vec{v}$ can be found by multiplying $\vec{v}$ by $\frac{1}{|\vec{v}|}$, therefore, $\frac{1}{|\vec{v}|}\vec{v}$ is a unit vector.

Any vector can be expressed as the product of its magnitude $|\vec{v}|$ and a unit vector $\hat{v}$ in the direction of $\vec{v}$,

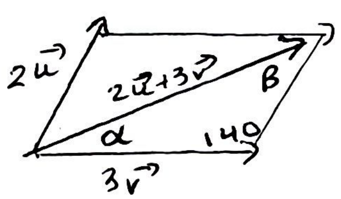
$$\vec{v} = |\vec{v}|\hat{v}.$$

NOTE: To find a unit $\hat{v}$, divide $\vec{v}$ by $|\vec{v}|$

Example: The vectors $\vec{u}$ and $\vec{v}$ are both unit vectors that make an angle of $40°$ to each other. Calculate the magnitude and direction of the following vectors. $\theta = 40°$ $\gamma = 180 - 40° = 140°$

a) $|2\vec{u} + 3\vec{v}| = \vec{x}$

* see pg 10 and 12



$|\vec{x}|^2 \doteq (2|\vec{u}|)^2 + (3|\vec{v}|)^2 - 2(2|\vec{u}|)(3|\vec{v}|)\cos 140$

$|\vec{x}|^2 \doteq 4 + 9 - 2(2)(3)\cos 140°)$

NOTE: unit vectors means the magnitude is 1

$|\vec{x}|^2 \doteq 22.19$

$|\vec{x}| \doteq 4.71$

$\frac{\sin\alpha}{2} = \frac{\sin 140°}{4.71}$

$\alpha \doteq 16°$

$\beta \doteq 180 - 140 - 16°$

$\doteq 24°$

b) $|4\vec{u} - 5\vec{v}| = \vec{y}$

$|\vec{y}|^2 \doteq 4^2 + 5^2 - 2(4)(5)\cos 40$

$|\vec{y}|^2 \doteq 10.36$

$|\vec{y}| \doteq 3.22$

$\frac{\sin\alpha}{4} = \frac{\sin 40}{3.22}$

$\alpha \doteq 53°$

$\beta = 180° - 40° - 53°$

$\doteq 87°$