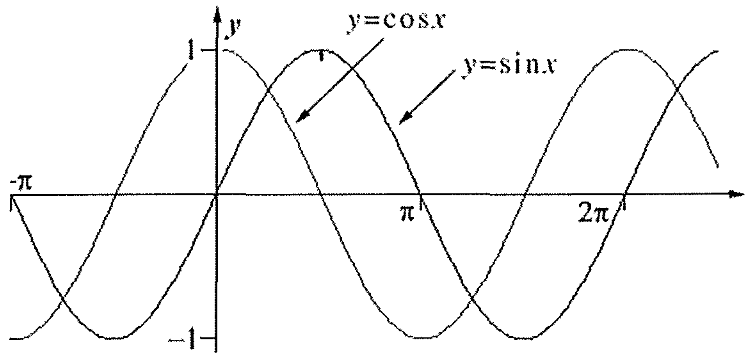
Day 4 – Equivalent Trigonometric Expressions

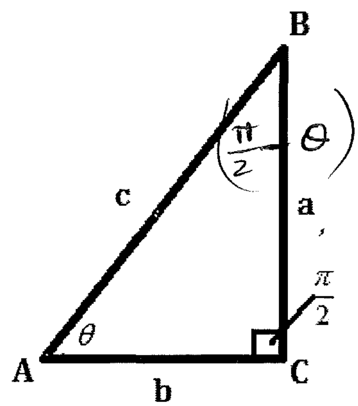
Investigation #1 – ***Recall:*** In grade 11 we learn about the graphs of $y = \sin x$ and $y = \cos x$. We can also graph these functions over radians, which we will see in the next unit. Below, we can see the graphs of both functions. What do you notice?

$\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$\cos\left(x - \frac{\pi}{2}\right) = \sin x$



Investigation #2 – See the triangle to the right:



1. Write an expression for $\sin\theta$ in terms of a, b, c.

   $\sin\theta = \frac{a}{c}$

2. Write expressions for sin, cos, and tan of angle B in terms of a, b, c and θ.

   $\sin\left(\frac{\pi}{2} - \theta\right) = \frac{b}{c}$

   $\cos\left(\frac{\pi}{2} - \theta\right) = \frac{a}{c}$

   $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{b}{a}$

3. What patterns do you see in the ratios?

   $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ $\qquad$ $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

   $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ $\qquad$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

   $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$ $\qquad$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

Given a trigonometric expression of a known angle, you can use **equivalent trigonometric expressions (co-function identities/complementary identities)** to evaluate expressions of other angles.

Cofunction/Complementary Identities			
$\sin x = \cos\left(\frac{\pi}{2} - x\right)$	$\cos x = \sin\left(\frac{\pi}{2} - x\right)$	$\sin\left(x + \frac{\pi}{2}\right) = \cos x$	$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$
$\tan x = \cot\left(\frac{\pi}{2} - x\right)$	$\cot x = \tan\left(\frac{\pi}{2} - x\right)$	$\tan\left(x + \frac{\pi}{2}\right) = -\cot x$	$\cot\left(x + \frac{\pi}{2}\right) = -\tan x$
$\csc x = \sec\left(\frac{\pi}{2} - x\right)$	$\sec x = \csc\left(\frac{\pi}{2} - x\right)$	$\csc\left(x + \frac{\pi}{2}\right) = \sec x$	$\sec\left(x + \frac{\pi}{2}\right) = -\csc x$

Steps to evaluating expressions using the Cofunction/complementary identities:

1. Determine what quadrant the given angle lies in (quadrant 1 or 2)
2. Break the angle up into terms of $\frac{\pi}{2}$
   - For Quadrant 1: $\theta = \frac{\pi}{2} - a$
   - For Quadrant 2: $\theta = \frac{\pi}{2} + a$
3. Use the corresponding complementary identify

EX 1 - Given that $\sin\frac{\pi}{5} \doteq 0.5878$, use equivalent trigonometric expressions to evaluate the following to four decimal places:

a) $\cos\frac{3\pi}{10}$

$\frac{3\pi}{10}$ is in Q1

$$\frac{3\pi}{10} = \frac{\pi}{2} - a$$

$$a = \frac{\pi}{2} - \frac{3\pi}{10}$$

$$= \frac{5\pi - 3\pi}{10} = \frac{2\pi}{10} = \frac{\pi}{5}$$

$$\therefore \cos\frac{3\pi}{10} = \sin\frac{\pi}{5} = 0.5878$$

b) $\cos\frac{7\pi}{10}$ ∈ Q2

$$\frac{7\pi}{10} = \frac{\pi}{2} + a$$

$$\frac{7\pi}{10} - \frac{\pi}{2} = a$$

$$\frac{7\pi - 5\pi}{10} = \frac{2\pi}{10} = \frac{\pi}{5} = a$$

$$\therefore \cos\left(\frac{7\pi}{10}\right) = -\sin\frac{\pi}{5}$$

$$= -0.5878$$

EX 2 – Given that $\cot\frac{4\pi}{9} = \tan\theta$,

a) Express $\frac{4\pi}{9}$ as a difference between $\frac{\pi}{2}$ and an angle

$$\frac{4\pi}{9} = \frac{\pi}{2} - a$$

$$a = \frac{\pi}{2} - \frac{4\pi}{9}$$

$$= \frac{9\pi - 8\pi}{18} = \frac{\pi}{18} \quad \therefore \cot\frac{4\pi}{9} = \tan\frac{\pi}{18}$$

b) Apply a co-function identity to determine the measure of angle θ

$$\theta = \frac{\pi}{18}. \qquad \text{Since} \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

EX 3 - Given that $\csc\beta = \sec 0.75$, and that β lies in the second quadrant, determine the measure of angle θ to two decimal places

In Q2: $\csc\beta = \sec\left(\beta + \frac{\pi}{2}\right)$ or $\csc\left(\beta + \frac{\pi}{2}\right) = \sec\beta$

In Q1: $\sec 0.75 = \csc\left(\frac{\pi}{2} - 0.75\right)$

$\csc(\theta) = \csc(\pi - \theta) \rightarrow$ according to CAST

$$= \csc\left(\pi - \left(\frac{\pi}{2} - 0.75\right)\right)$$

$$= \csc\left(\frac{\pi}{2} + 0.75\right)$$

$$\therefore \theta = \frac{\pi}{2} + 0.75 \doteq 2.32$$

$\therefore$