Day 4: 6.3 – Transformations of Logarithmic Functions

Warm up:

1. Rewrite in logarithmic form: $8 = 2^3$ $\log_2 8 = 3$

2. Rewrite in exponential form: $\log_4 16 = 2$ $\Rightarrow$ $4^2 = 16$

3. Evaluate the logarithms:

   a. $\log_3\left(\frac{1}{27}\right) = x \Rightarrow 3^x = \frac{1}{27}$

   $\boxed{x = -3}$

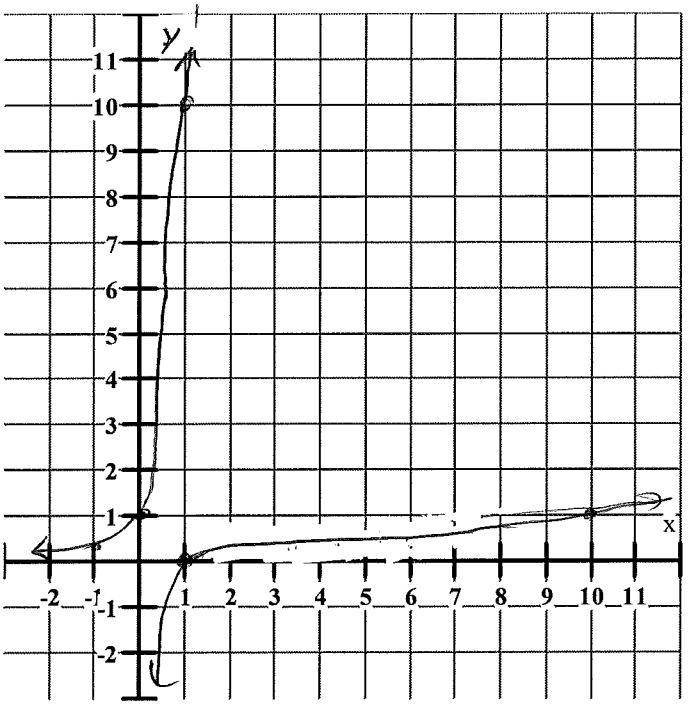
   b. $\log 0.0001 = x$

   $10^x = \frac{1}{10000}$

   $10^x = 10^{-4}$

   $x = -4$

4. Graph and label each of the following functions on the following grid: $f(x) = 10^x$ and $f^{-1}(x)$



Recall:
A logarithm is the inverse of an exponential function.

- If $x = 10^y$, then $y = \log_{10} x$

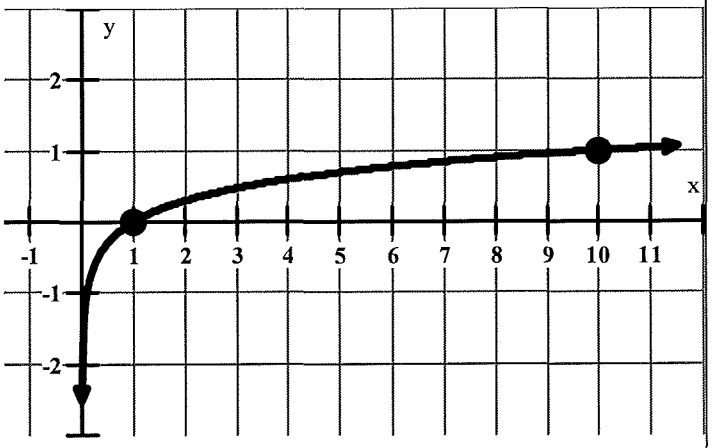
Recall:
Logarithms to the base 10 are called **common logarithms**.

- log 100 is the same as $log_{10}100$

$f(x) = \log x$ is our new **parent function** that we can transform like any parent function, in the form:

$$y = a\log[k(x-d)]+c$$

- a represents a **vertical stretch/compression** and/or **reflection in the x-axis**
- k represents a **horizontal stretch/compression** and/or **reflection in the y-axis**
- d represents a **horizontal shift**
- c represents a **vertical shift**



- ***When graphing, use key points*** (1, 0), (10, 1), ***and the*** (0.1, -1)

- ***Graph in the order:***
  1. ***Stretch/Compress/Reflect***
  2. ***Translate***

EX 1 - Write an equation for a common logarithm function that is:

- Vertically compressed by factor of $\frac{1}{2}$
- Reflected in the y-axis
- Horizontally stretched by factor of 3
- Vertically shifted down 5 units

$y = \frac{1}{2}\log\left[-\frac{1}{3}x\right] - 5$

EX 2 – List the transformations, in order, applied to the parent function $f(x) = \log x$ to arrive at the following functions.

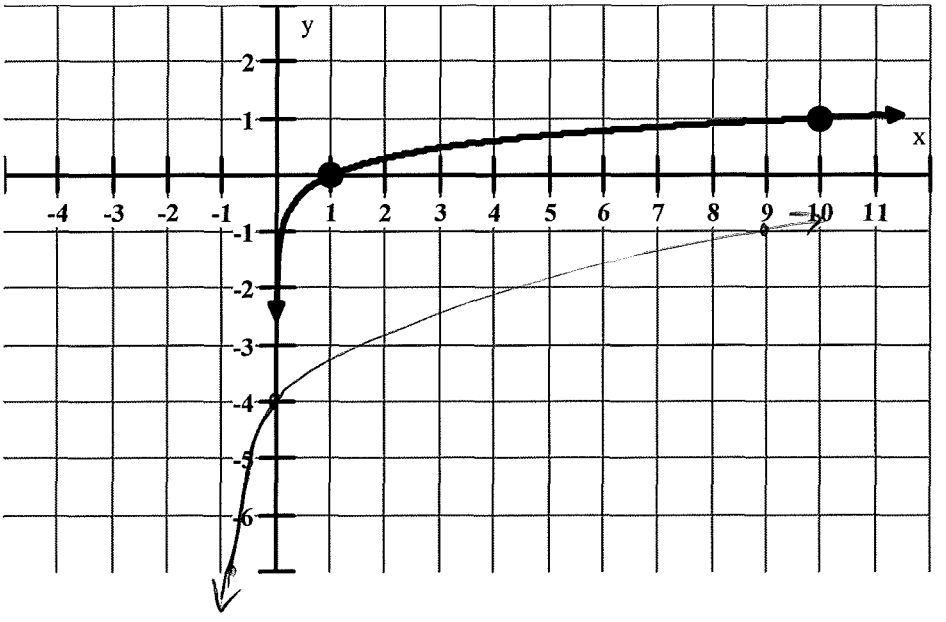
$g(x) = 3\log(4x - 8)$

$= 3\log[4(x-2)]$

- vertically stretched by a factor of 3
- horizontally compressed by a factor of 1/4
- translation 2 units to the right

EX 3 – Graph and state key properties for each function. **Include the parent function.**

a) $f(x) = 3\log(x+1) - 4$



$(x,y) \to (x-1, 3y-4)$

$(0.1, -1) \to (-0.9, -7)$

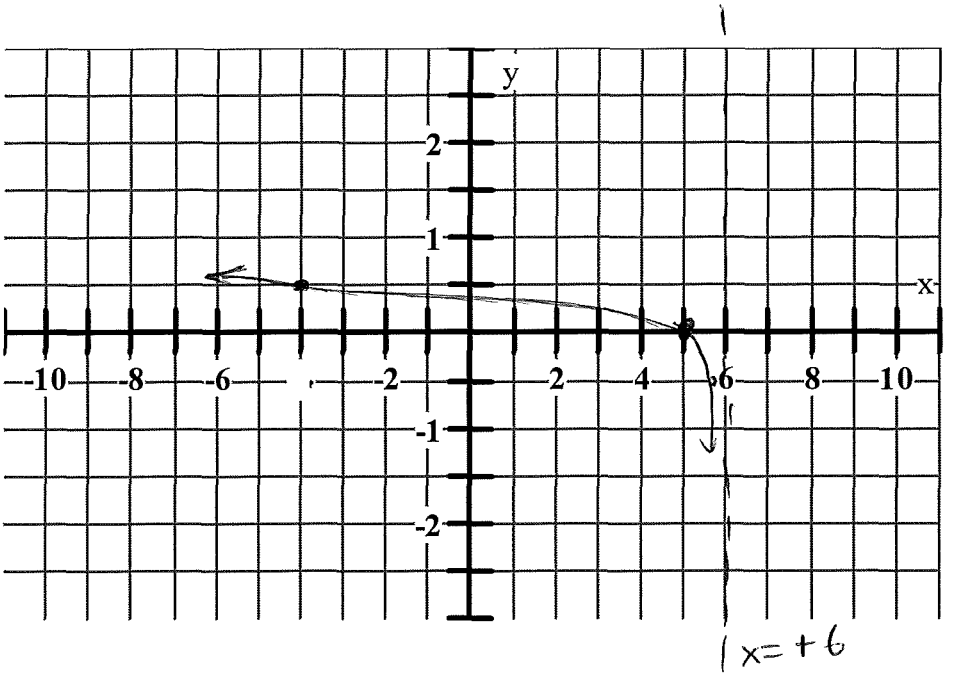
$(1, 0) \to (0, -4)$

$(10, 1) \to (9, -1)$

Domain: $\{x \in \mathbb{R} \mid x > -1\}$ Range: $\{y \in \mathbb{R}\}$

Asymptote: $x = -1$

b) $h(x) = \frac{1}{2}\log(-x+6) = \frac{1}{2}[-(x-6)]$ $\quad (x,y) \to (-x+6, \frac{1}{2}y]$



$(0.1, -1) \to (5.9, -\frac{1}{2})$

$(1, 0) \to (5, 0)$

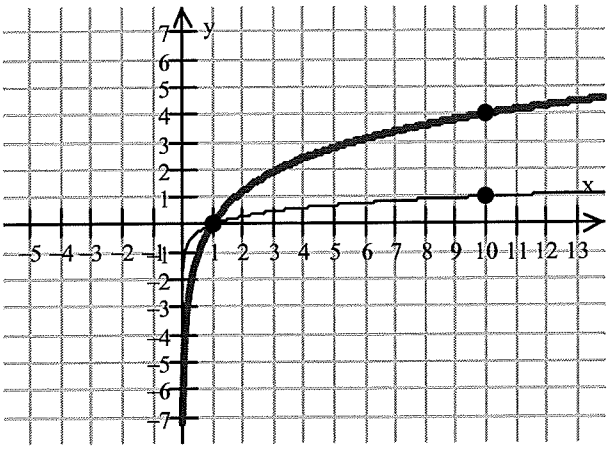
$(10, 1) \to (-4, \frac{1}{2})$

Domain: $\{x \in \mathbb{R} \mid x < 6\}$ Range: $\{y \in \mathbb{R}\}$

Asymptote: $x = 6$

EX 4: Write the **equation** for each graph shown below.

a)

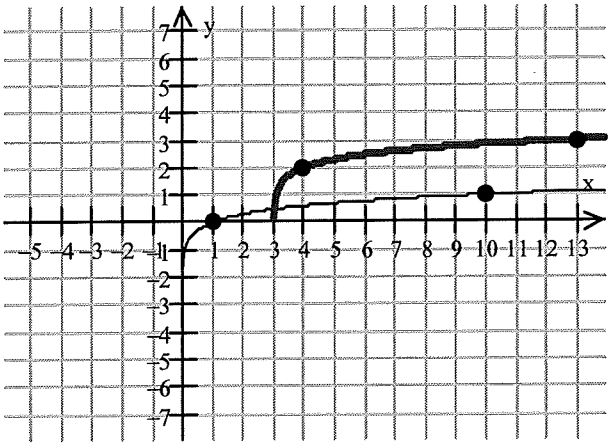


$(10,1) \rightarrow (10,4)$

Vertically stretched by a factor of 4

$\therefore \; y = 4 \log x$

b)



$(1,0) \Rightarrow (4,2)$

$(10,1) \rightarrow (13,3)$

3 Right and 2 up

$y = \log(x-3) + 2$