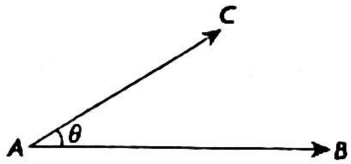
Day 3: 7.3 The Dot Product of 2 Geometric Vectors

The **Dot product** of two geometric vectors



$$\overrightarrow{AC} \cdot \overrightarrow{AB} = |\overrightarrow{AC}||\overrightarrow{AB}|\cos\theta, 0 \leq \theta \leq 180°$$

The dot product of two vectors is a scalar (also called the scalar product).

> For the vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta, 0 \leq \theta \leq 180°$:
> - for $0 \leq \theta < 90°$, $\cos\theta > 0$, so $\vec{a} \cdot \vec{b} > 0$
> - for $\theta = 90°$, $\cos\theta = 0$, so $\vec{a} \cdot \vec{b} = 0$
> - for $90° < \theta \leq 180°$, $\cos\theta < 0$, so $\vec{a} \cdot \vec{b} < 0$

Properties of the Dot Product:

1. Two nonzero vectors are perpendicular. *if* $\vec{a} \perp \vec{b}$ *then* $\vec{a} \cdot \vec{b} = 0$
2. Commutative property: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.
3. Associative property with a scalar *k*: $(k\vec{a}) \cdot \vec{b} = \vec{a} \cdot (k\vec{b}) = k(\vec{a} \cdot \vec{b})$.
4. Distributive property: $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
5. Magnitudes property: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

Ex1: Calculate $\vec{u} \bullet \vec{v}$ if $|\vec{u}| = 10$, $|\vec{v}| = 30$ and $\theta = 32°$.

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$$
$$= (10)(30)\cos 32°$$
$$\approx 254.41$$

Ex2: Calculate the angle between $\vec{u}$ *and* $\vec{v}$ given $|\vec{u}| = 7$, $|\vec{v}| = 5$ *and* $\vec{u} \bullet \vec{v} = 17.5$.

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$$
$$17.5 = (7)(5)\cos\theta$$
$$\cos\theta = \frac{17.5}{35} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \theta$$
$$\cos\theta = 1/2 \qquad \theta = 60°$$

NOTE: $\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\cos 0$
$= |\vec{a}|^2$

Ex3: Expand and simplify:

a) $(k\vec{u})\bullet(\vec{u}+\vec{v})$

$= k(\vec{u}\cdot\vec{u}) + k(\vec{u}\cdot\vec{v})$

$= k(|\vec{u}|^2) + k(\vec{u}\cdot\vec{v})$

b) $(\vec{a}+\vec{b})\bullet(\vec{a}-\vec{b})$

$= \vec{a}\cdot\vec{a} - \cancel{\vec{a}\cdot\vec{b}} + \cancel{\vec{a}\cdot\vec{b}} - \vec{b}\cdot\vec{b}$

$= |\vec{a}|^2 - |\vec{b}|^2$

c) $(3\vec{a}+4\vec{b})\bullet(5\vec{a}+6\vec{b})$

$= 15\,\vec{a}\cdot\vec{a} + 18\,\vec{a}\cdot\vec{b} + 20\,\vec{a}\cdot\vec{b} + 24\,\vec{b}\cdot\vec{b}$

$= 15\,|\vec{a}|^2 + 38\,\vec{a}\cdot\vec{b} + 24\,|\vec{b}|^2$

Application of the Dot Product:

Work is done when a **force** acting on an object causes a displacement of an object from one position to another. **Work** is a scalar quantity measured in joules (J).

Work is defined as the dot product: $W = \vec{F}\bullet\vec{s}$ *or* $W = |\vec{F}||\vec{s}|\cos\theta$ where $\vec{F}$ is the force acting on an object (N), $\vec{s}$ is the displacement caused by the force (m) and θ is the angle between $\vec{F}$ *and* $\vec{s}$.

Ex5: A 25 kg box is located 8 m up a ramp inclined at an angle of $18°$ to the horizontal. Determine the work done by the force of gravity as the box slides to the bottom of the ramp.

$|\vec{F}| = (25)(9.8) = 245$

NOTE: θ will be 90-18

$W = |\vec{F}||\vec{s}|\cos\theta$

$= (245)(8)\cos(72°)$

$= 605.7$ Joules