Day 3: 6.2 Vector Addition & Subtraction

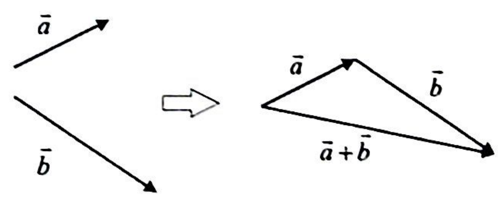
Sum (Resultant) of 2 vectors:

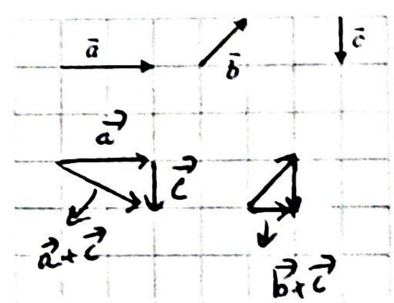
The *vector addition* $\vec{s}$ of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} + \vec{b}$ and is called the *sum* or *resultant* of the two vectors. So: $\vec{s} = \vec{a} + \vec{b}$

Triangle Rule (Tail to Tip/Head Rule)
In order to find the sum (resultant) of two geometric vectors:
a) Place the second vector with its tail on the tip (head) of the first vector.
b) The sum (resultant) is a vector with the tail at the tail of the first vector and the head at the head of the second vector.

Ex 1. Use the following diagram and the triangle rule to compute the required operations.

a) $\vec{a} + \vec{b}$, b) $\vec{a} + \vec{c}$, c) $\vec{b} + \vec{c}$

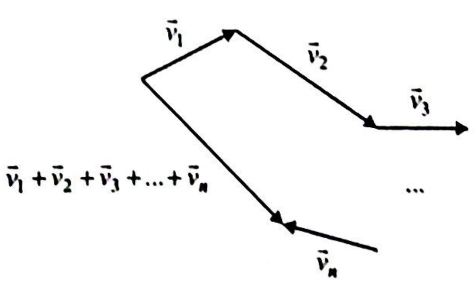


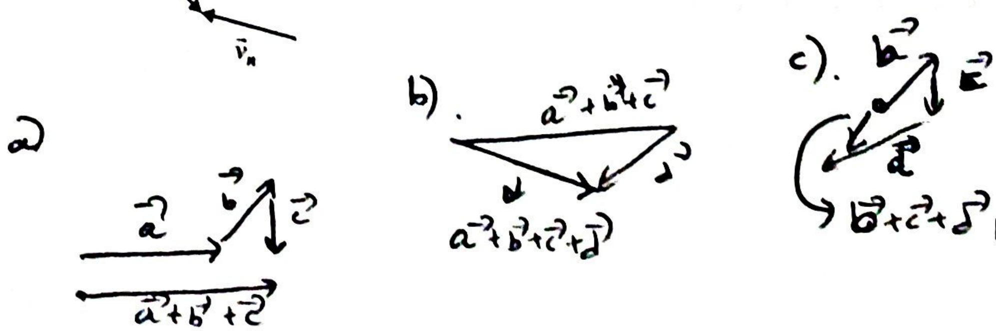


C Polygon Rule
In order to find the sum (resultant) of *n* geometric vectors:
a) Place the next vector with its tail on the tip (head) of the precedent vector.
b) The sum (resultant) is a vector with the tail at the tail of the first vector and the head at the head of the last vector

Ex 2: Use the following diagram and the triangle rule to compute the required operations

a) $\vec{a} + \vec{b} + \vec{c}$, b) $\vec{a} + \vec{b} + \vec{c} + \vec{d}$, c) $\vec{b} + \vec{c} + \vec{d}$





D Parallelogram Rule (Tail to Tail Rule)

To add two geometric vectors, the following rule can also be used:

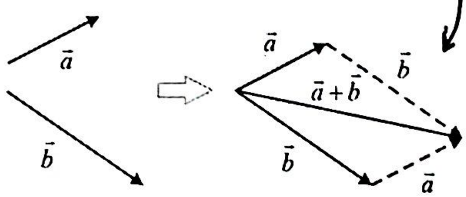
a) Position both vectors with their *tails* at the same point.

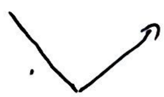
b) Build a *parallelogram* using the vectors as two sides.

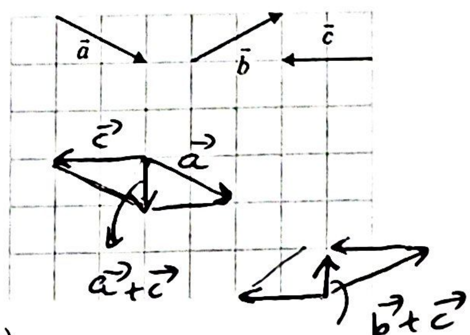
c) The sum (resultant) is the *diagonal* of the parallelogram starting from the common tail point.

Ex 3. Use the parallelogram rule to compute the required operations:

a) $\vec{a}+\vec{b}$, b) $\vec{a}+\vec{c}$, c) $\vec{b}+\vec{c}$ a)







Using the diagram below to answer the questions a) to g)

a) $\overrightarrow{AB}+\overrightarrow{BC}$

$=\overrightarrow{AC}$

b) $\overrightarrow{CD}+\overrightarrow{DA}$

$=\overrightarrow{CA}$

c) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$

$=\overrightarrow{AD}$

d) $\overrightarrow{CD}+\overrightarrow{DA}+\overrightarrow{AB}$

$=\overrightarrow{CB}$

e) $\overrightarrow{BC}+\vec{0}$

$=\overrightarrow{BC}$

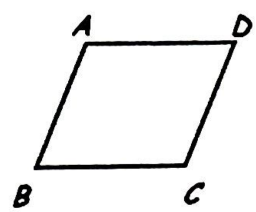
f) $\overrightarrow{AB}+\overrightarrow{CD}$

$=\overrightarrow{AB}-\overrightarrow{DC}$

$=\overrightarrow{AB}-\overrightarrow{AB}$

$=\vec{0}$

$\boxed{\overrightarrow{AB}=\overrightarrow{DC}}$



g) $\overrightarrow{AB}+\overrightarrow{DC}$

$=\overrightarrow{AB}+\overrightarrow{AB}$

$=2\overrightarrow{AB}$

Can we answer the following without any diagram? Yes

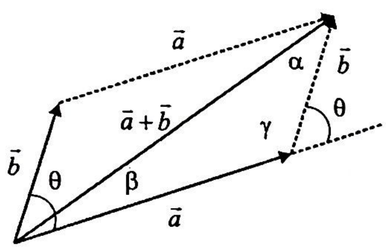
a) $\overrightarrow{AB} + \overrightarrow{BC}$ b) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CF}$ c) $\overrightarrow{CD} + \overrightarrow{DH} + \overrightarrow{HF}$

$= \overrightarrow{AC}$ $= \overrightarrow{AF}$ $= \overrightarrow{CF}$

please NOTE: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ → we start and finish at the same point

Magnitude and Direction for Vector Sum

Let $\theta = \sphericalangle(\vec{a}, \vec{b})$ be the angle between the vectors $\vec{a}$ and $\vec{b}$ when they are placed tail to tail.



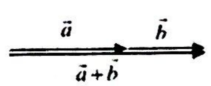
$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\gamma \qquad \gamma = 180^o - \theta$$

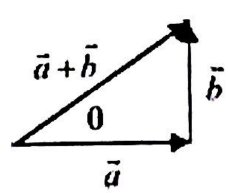
To find the angle between the sum vector and the component vectors we use:

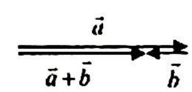
$$\frac{|\vec{a}|}{\sin\alpha} = \frac{|\vec{b}|}{\sin\beta} = \frac{|\vec{a}+\vec{b}|}{\sin\gamma}$$

Ex 4: Special case:

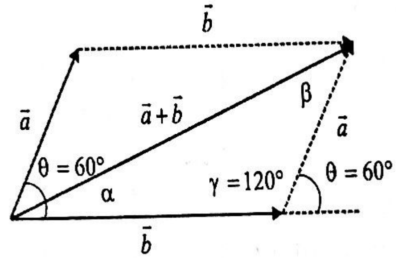
$\theta = 0$ $\theta = \frac{\pi}{2}$ $\theta = \pi$







Ex 5: Given the magnitude of two vectors $|\vec{a}| = 4$ *and* $|\vec{b}| = 7$, and the angle between them when placed tail to tail as being $\theta = 60°$, find the magnitude of the vector sum $\vec{s} = \vec{a} + \vec{b}$ and the direction (the angles between the vector sum and each vector).



$$|\vec{s}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos 120°$$

$$= 4^2 + 7^2 - 2(4)(7)\cos(120°)$$

$$|\vec{s}|^2 = 93$$

$$|\vec{s}| = \sqrt{93}$$

$$\frac{\sin\alpha}{\cancel{|a|}4} = \frac{\sin 120}{\sqrt{93}}$$

$$\alpha = 21°$$

$$\beta = 180 - 120° - 21° = 59°$$

Ex 6: Given the magnitude of two vectors $|\vec{a}| = 4$, $|\vec{b}| = 6$ *and* $|\vec{a} + \vec{b}| = 8$, find the angle between them when placed tail to tail $\theta = \angle(\vec{a}, \vec{b})$ Let $\gamma = 180 - \theta$

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\gamma.$$

$$8^2 = 4^2 + 6^2 - 2(4)(6)\cos(180-\theta).$$

$$64 = 52 - 48\cos(180-\theta) \Rightarrow \cos(180-\theta) = \frac{64-52}{-48}$$

$$\cos(180-\theta) = -\frac{1}{4} \Rightarrow 180-\theta = 104°$$

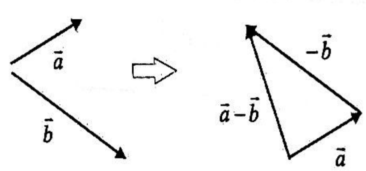
$$\theta = 76°$$

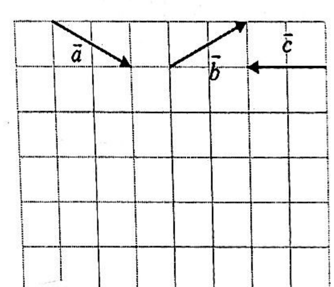
Vector Subtraction

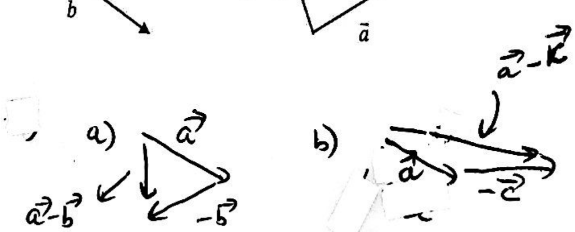
understood as a vector addition between the first vector and the opposite of the second vector

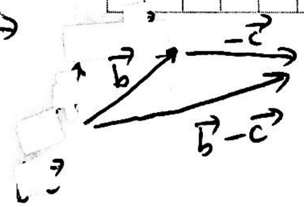
The subtraction operation between two vectors can be $\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$:

Ex 7: Find $\quad a)\ \vec{a} - \vec{b},\quad b)\ \vec{a} - \vec{c},\quad c)\ \vec{b} - \vec{c}$



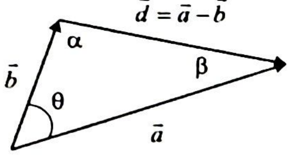






Magnitude and Direction for Vector Difference

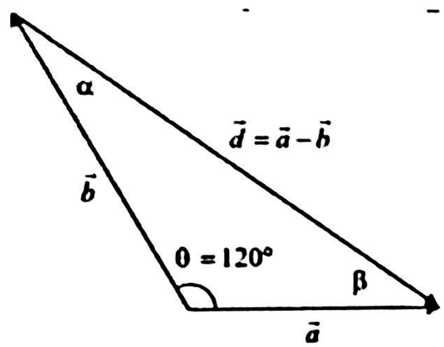
Let θ be the angle between the vectors *a* and *b* when they are placed tail to tail, the magnitude of the *vector difference* is given by:



$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$\frac{|\vec{a}|}{\sin\alpha} = \frac{|\vec{b}|}{\sin\beta} = \frac{|\vec{a}-\vec{b}|}{\sin\theta}$$

Ex 8: Given the magnitude of two vectors $|\vec{a}| = 10$ *and* $|\vec{b}| = 14$, and the angle between them when placed tail to tail as being θ = 120°. Find the magnitude of the difference vector $\vec{d} = \vec{a} - \vec{b}$ and the direction (the angles between the difference vector and each vector).



$|\vec{d}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

$|\vec{d}|^2 = 10^2 + 14^2 - 2(10)(14)\cos 120°$

$|\vec{d}|^2 = 436$

$|\vec{d}| = \sqrt{436} = 2\sqrt{109}$

$\frac{\sin\alpha}{10} = \frac{\sin 120°}{2\sqrt{109}} \Rightarrow \alpha \doteq 25°$

$\beta = 180° - 120° - 25°$

$= 35°$

Ex 9: Express in term of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$

a) $\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$

$= \vec{a} + \vec{c}$

b) $\overrightarrow{CF} = \overrightarrow{CB} + \overrightarrow{BF} = -\vec{b} + \vec{c}$

$= \vec{c} - \vec{b}$

c) $\overrightarrow{AG} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CG} = \vec{a} + \vec{b} + \vec{c}$

d) $\overrightarrow{BH} = \overrightarrow{BA} + \overrightarrow{AE} + \overrightarrow{EH}$

$= -\vec{a} + \vec{c} + \vec{b}$

$= \vec{b} + \vec{c} - \vec{a}$