2.2 – The Factor Theorem

Day 1 – Integral Zero Theorem

The Factor Theorem *(an extension of the remainder theorem)*

$x-b$ is a factor of polynomial $P(x)$ if and only if $P(b)=0$

$ax-b$ is a factor of polynomial $P(x)$ if and only if $P\left(\frac{b}{a}\right)=0$

Example One – Check: Are $x-3$ and $x+2$ factors of $P(x)=x^3-x^2-14x+24$?

$P(3)=3^3-3^2-14(3)+24 = 27-9-42+24=0 \Rightarrow x-3$ is a factor

$P(-2)=(-2)^3-(-2)^2-14(-2)+24=-8-4+28+24\neq 0 \Rightarrow x+2$ is NOT a factor.

The Integral Zero Theorem *(leading coefficient is 1)*

If $x-b$ is a factor of a polynomial $P(x)$ with leading coefficient 1, then

- **b is a factor of the constant term of $P(x)$**

* *Once one factor of a polynomial is found, division can be used to determine other factors.*

Example Two – Factor x^3+2x^2-5x-6 // $P(x)$

* **Let** $P(x)$ = the function

Step 1) List factors of the constant term.

Step 2) Test factors into the function $P(x)$.

Step 3) Use long synthetic to find quotient $Q(x)$

Step 4) Factor $Q(x)$

b: $\pm1, \pm2, \pm3, \pm6$

$P(1)=1+2-5-6\neq 0$

$P(-1)=-1+2+5-6=0$

$\therefore x+1$ is a factor.

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array} \rightarrow \text{Rem}=0$$

you may also use long division

$\therefore P(x)=(x+1)(x^2+x-6)$

$=(x+1)(x+3)(x-2)$.

Example Three – Factor $x^4+3x^3-7x^2-27x-18$

$b: \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$P(1) = 1+3-7-27-18 \neq 0$

$P(-1) = 1-3-7+27-18 = 0 \Rightarrow x+1$ is a factor.

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & -7 & -27 & -18 \\ & \downarrow & -1 & -2 & 9 & 18 \\ \hline & 1 & 2 & -9 & -18 & 0 \end{array}$$

$\therefore P(x) = (x+1)(x^3+2x^2-9x-18)$

we need to find one more 'b' such that $P(b)=0$

$P(2) \neq 0 \qquad P(-2) = 0 \Rightarrow (x+2)$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -9 & -18 \\ & \downarrow & -2 & 0 & 18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$\therefore P(x) = (x+1)(x+2)(x^2-9)$

$= (x+1)(x+2)(x-3)(x+3)$

NOTE: For $x^3+2x^2-9x-18$, grouping would also have worked.

$x^3+2x^2-9x-18$

$= x^2(x+2)-9(x+2)$

$= (x^2-9)(x+2)$