Day 3: 1.1-Power Functions

Polynomial Function: a series of terms in which each term is the product of a constant and a power of x that has a whole number as the exponent. Polynomial functions have the form:

$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots a_3 x^3 + a_2 x^2 + a_1 x + a_0$, where a is a constant and n is a whole number.

Example One: Which of the following represent polynomial functions?

a) $f(x) = 7x^3 + 5x^2 + 2$

Polynomial

b) $f(x) = x^{-4}$

not a polynomial

c) $g(x) = -\cos x$

Not a polynomial

d) $y = 8^x$

not a polynomial

e) $y = -7$

Polynomial

f) $f(x) = -3x^5$

Polynomial

A **power function** is the simplest type of polynomial function and has the form $y = ax^n$ Where x is a variable, a is a real number, and n is a whole number.

Polynomial functions are named based on their **degree.**

Degree of 0: $y = a$

1: $y = ax + b,\ a \neq 0$

2: $y = ax^2 + bx + c,\ a \neq 0$

3: $y = ax^3 + bx^2 + cx + d$
$a \neq 0$

4: $y = ax^4 + bx^3 + cx^2$
$+ dx + e,\ a \neq 0$

5: $y = ax^5 + bx^4 + cx^3 + dx^2$
$+ ex + f$

Key Terms:

Degree: is the exponent of the greatest power of x

Leading coefficient: is the coefficient of the greatest power of x

Constant Term: a term without a variable

Example Two: State the degree, leading coefficient and constant term for the following:

	Degree	**Leading Coefficient**	**Constant Term**
$y = x^4 - 3x^2 + 6$	4	1	6
$f(x) = 3x^3 - \frac{5}{2}x^5$	5	$-5/2$	0
$g(x) = 4$	0	4	4

Interval Notation:

In grade 11 you used set notation to describe the domain and range of a function. **Interval notation** is another way to express this using brackets to represent intervals.

Set Notation	Interval Notation
$\{x \in R \mid -2 < x \leq 6\}$	(-2, 6]

Note:

- Intervals that are infinite are expressed using the infinity symbol (∞) or negative infinity symbol ($-\infty$)
- **Square** brackets indicate that the end value **is included** in the interval (*Replaces* $\leq, \geq$)
- **Round** brackets indicate that the end value is **not included** (*Replaces* $<, >$)
- **Round** brackets are **always** used at positive or negative **infinity**

Example Three: Rewrite the following in interval notation

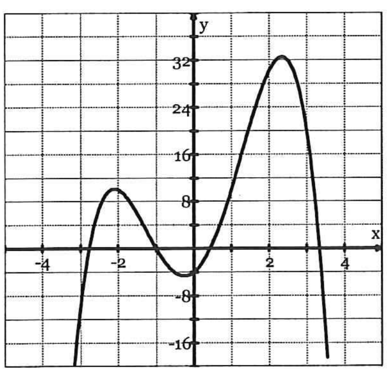
$\{x \in R \mid x \leq -1\}$	$(-\infty, -1]$
x is greater than a and less than or equal to b	$[a, b]$
$\{x \in R\}$	$(-\infty, \infty)$

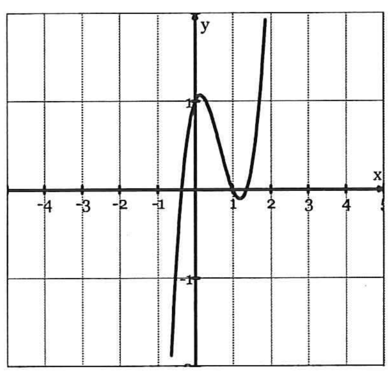
End Behaviour: describes the behavior of the y-values as x increases and as x decreases.

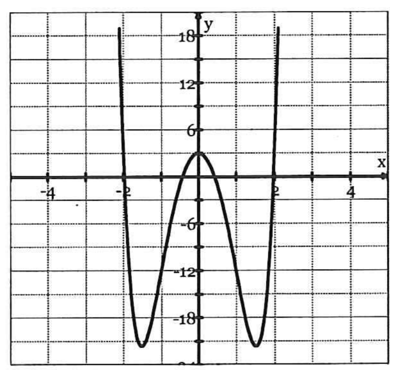
It can be described in two ways:

$as\ x \to \infty, y \to \infty$ $as\ x \to -\infty, y \to -\infty$	**OR**	Extends from quadrant 2 to quadrant 4 $Q2 \to Q4$

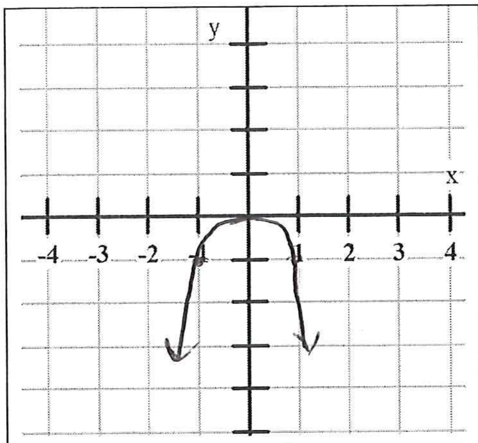
Example Four: State the end behavior of the following functions in two ways.



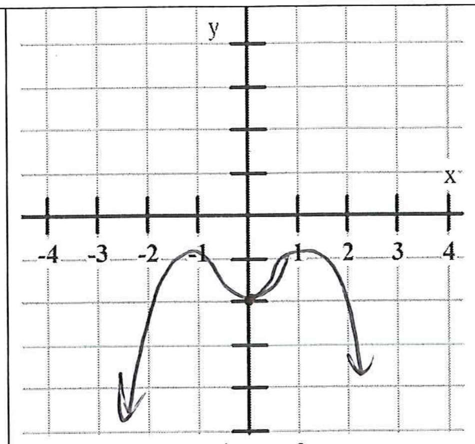




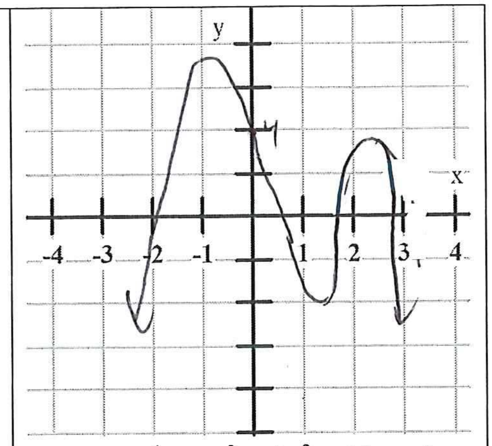
$x \to \infty \quad y \to -\infty$ $x \to -\infty \quad y \to -\infty$	$x \to \infty \quad y \to \infty$ $x \to -\infty \quad y \to -\infty$	$x \to \infty \quad y \to \infty$ $x \to -\infty \quad y \to \infty$
$Q3 \to Q4$	$Q1 \to Q3$	$Q1 \to Q2$



$y = -x^4$



$y = -x^4 + 2x^2 - 2$



$y = -x^4 + 3x^3 + 3x^2 - 11x + 4$

What can we assume about even-degree functions based on the graphs above?

End behavior	$x \to \infty$ $y \to \infty$, $x \to -\infty$ $y \to \infty$ or $x \to \infty$ $y \to -\infty$, $x \to -\infty$ $y \to -\infty$
x – intercepts	can have 0 x-intercepts (up to n x-intercepts)
Global maxima/minima	will always have a global max/min depending on the sign of leading coefficient.
Turning points	at least 1 turning point or 3 or 5,... max n-1

In summary:

Odd-Degree Functions		***Even-Degree Functions***	
Positive leading coefficient	$x \to \infty$ $y \to \infty$, $x \to -\infty$ $y \to -\infty$	**Positive leading coefficient**	$x \to \infty$ $y \to \infty$, $x \to -\infty$ $y \to \infty$
Negative leading coefficient	$x \to \infty$ $y \to -\infty$, $x \to -\infty$ $y \to \infty$	**Negative leading coefficient**	$x \to \infty$ $y \to -\infty$, $x \to -\infty$ $y \to -\infty$
Number of x – intercepts	at least one, max n	**Number of x – intercepts**	0 (min) n (max)
Number of absolute max/min points	None	**Number of absolute max/min points**	absolute min $a > 0$ absolute min $a < 0$

All polynomial functions with a degree *n*, will have at most n-1 **turning points.**