Day 3: MCR3U1

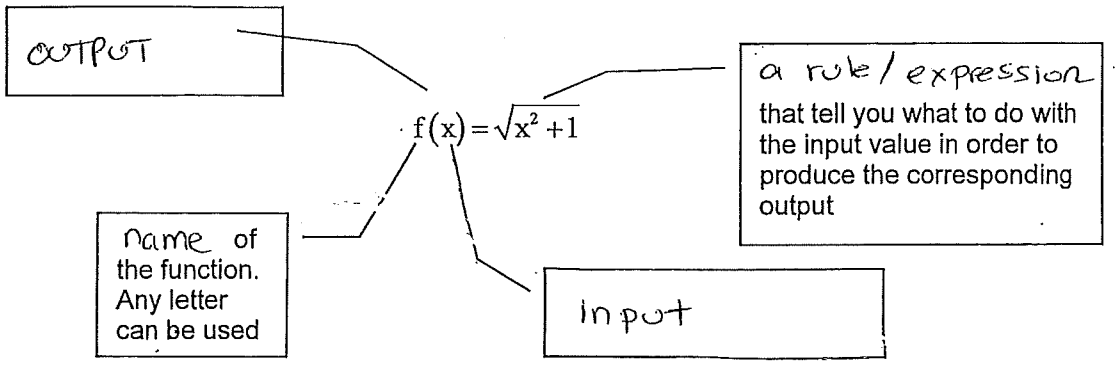
1.2 Function Notation

Because functions are used throughout mathematics, f(x) notation, was developed to make it easier to work with them. Function notation can be used even when we do not know the details of a particular relationship.

Suppose a function is given. Let f denote a given function and let x represent the input value.

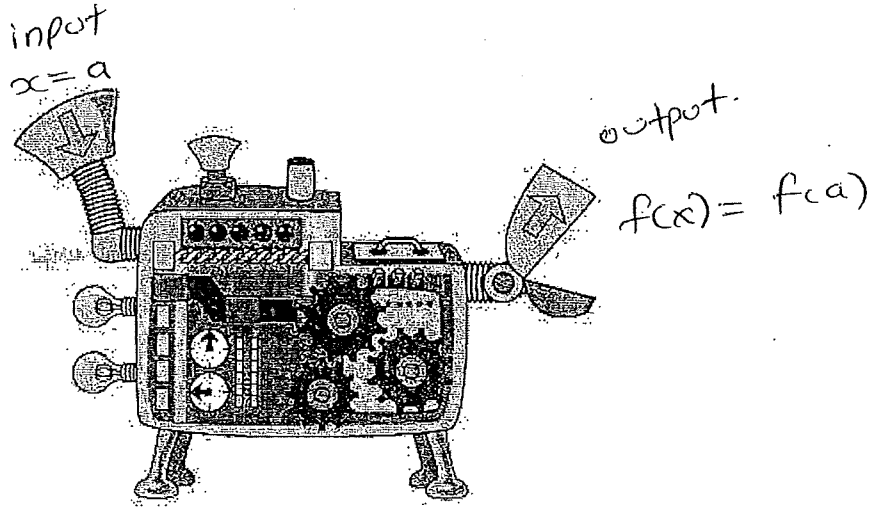
f(x) denotes the value (output)of the function when the input is x. In other words, $f(x)=y$.

Usually a function is represented by a mathematical expression.



* **Don't confuse f(x) with multiplication!** The entire symbol f(x) represents a function, its input, and its output. It is not the same as an algebraic expression.

The Function Machine:



Note: $f(a+b) \neq f(a)+f(b)$

Evaluating Functions

Example 1 If f(x) = 2x + 3, find

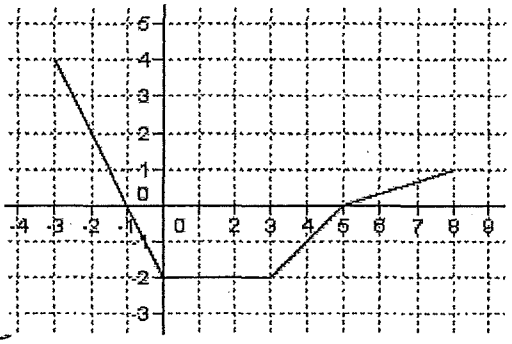
a) $f(6) = 2(6)+3$
$= 12+3$
$= 15$

b) $f(-5) = 2(-5)+3$
$= -10+3$
$= -7$

c) $f(x+1)$
$= 2(x+1)+3$
$= 2x+2+3$
$= 2x+5$

d) $f(2x) = 2(2x)+3$
$= 4x+3$

Example 2
Given the graph to the right, find



a) $f(2) = -2$ b) $f(-3) = 4$

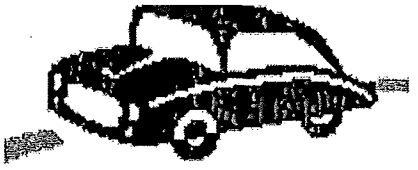
c) x if $f(x) = 2$
$x = -2$

d) x if $f(x) = 0$
$x = -1$ or $x = 5$

Application

A company rents cars for $50 per day plus $0.15/km.

a) Express the daily rental cost, C as a function of the number of kilometres, d travelled. $C(d) = 0.15d + 50$



b) Determine the rental cost if you drive 472 km in one day.

$C(472) = 0.15(472)+50$
$= \$120.80$ ∴ Rental cost would be $120.80 for 472 km.

c) Determine how far you can drive in a day for $80.

$80 = 0.15d + 50$
$0.15d = 80-50$ → $0.15d = 30$
$d = 200$ ∴ 200 km driven

d) Is C(d) a function? Justify your answer.

Yes. Linear line which passes VLT.

1.4 Domain and Range

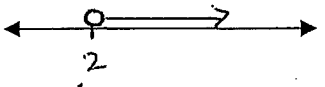
Number Systems:

Type of Number	Quick Description
Counting Numbers $\mathbb{N}$	{1, 2, 3, ...}
Whole Numbers $\mathbb{W}$	{0, 1, 2, 3, ...}
Integers $\mathbb{Z}$	{..., -3, -2, -1, 0, 1, 2, 3, ...}
Rational Numbers $\mathbb{Q}$	p/q : p and q are integers, q is not zero
Irrational Numbers $\overline{\mathbb{Q}}$	Not Rational
Real Numbers $\mathbb{R}$	Rationals and Irrationals

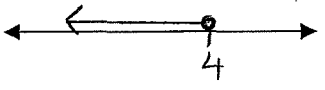
Inequalities

Symbols:

>: greater than Gator is greater ☺
<: less than
≥: greater than or equal to
≤: less than or equal to
o: open circle, does not include point Ex. x > 2:



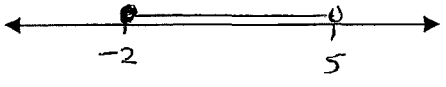
●: closed circle, includes point Ex a ≤ 4:



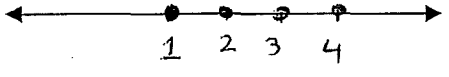
{ }: set
|: such that
∈: an element of, or in the set of

Graphing on a Number Line

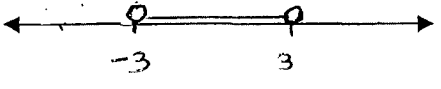
Graph { x ∈ R | -2≤x<5 }



Graph { x ∈ N | -2≤x<5}



Graph { x ∈ R |$x^2 < 9$}



$-3 < x < 3$

Domain: the set of all values of the independent variable (x) of a relation

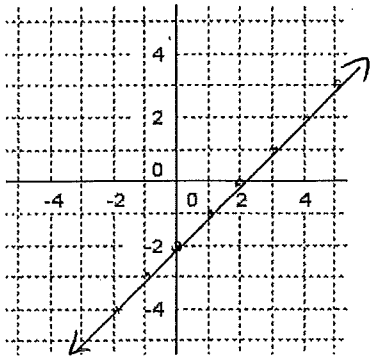
Range: the set of all values of the dependent variable (y) of a relation.

Notation: { ∈ | }

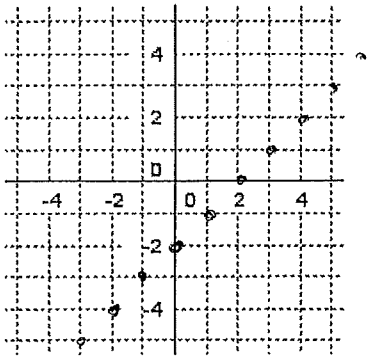
variable number system condition

Example 1 Graph the following functions below:

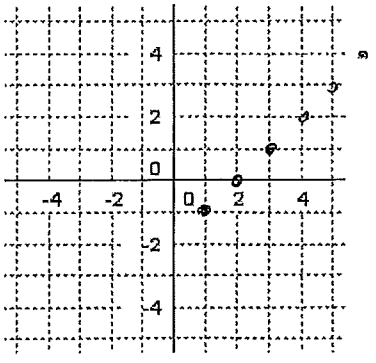
$y = x - 2, D = \{x \in \mathbb{R}\}$



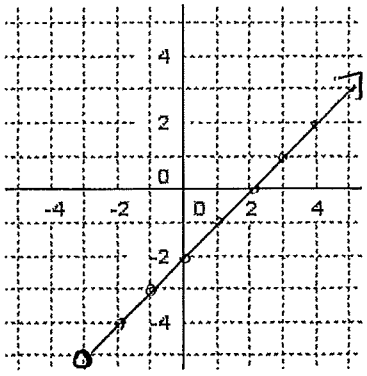
$y = x - 2, D = \{x \in \mathbb{Z}\}$



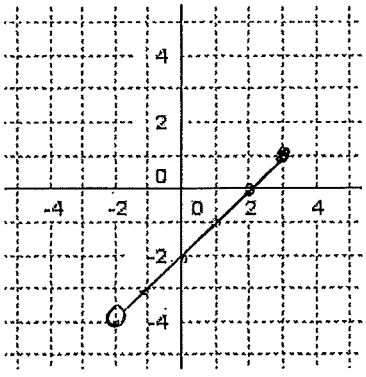
$y = x - 2, D = \{x \in \mathbb{N}\}$



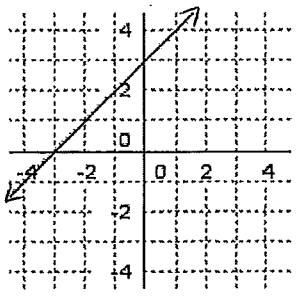
$y = x - 2, D = \{x \in \mathbb{R} | x > -3\}$



$y = x - 2, D = \{x \in \mathbb{R} | -2 < x \leq 3\}$

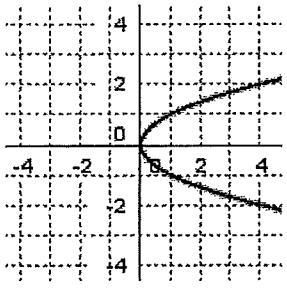


Example 2 State the domain and range for the following.



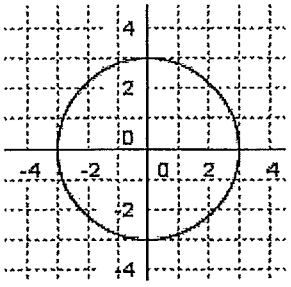
$\{x \in \mathbb{R}\}$

$\{y \in \mathbb{R}\}$



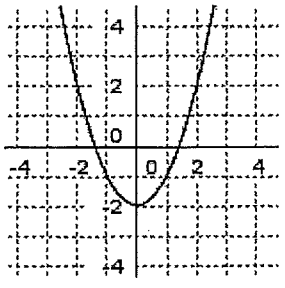
$\{x \in \mathbb{R} | x \geq 0\}$

$\{y \in \mathbb{R}\}$

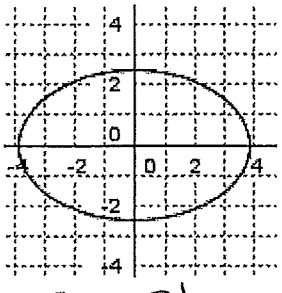


$\{x \in \mathbb{R} | -3 \leq x \leq 3\}$

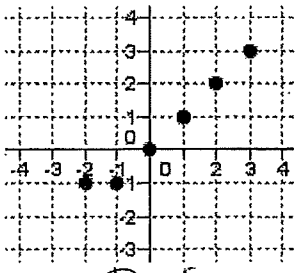
$\{y \in \mathbb{R} | -3 \leq y \leq 3\}$



$\{x \in \mathbb{R}\}$



$\{x \in \mathbb{R} | -4 \leq x \leq 4\}$



$D = \{-2, -1, 0, 1, 2,$

Example 3 Write the domain and range for the following relations.

a) $\{(1,3),(1,4),(1,5),(1,6)\}$

$D = \{1\}$

$R = \{3,4,5,6\}$

b) $y = \sqrt{x}$

$\sqrt{\text{negative}} = \text{ERROR}$

$D = \{x \in \mathbb{R} \mid x \geq 0\}$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

c) $y = \frac{1}{x}$

$\frac{1}{0} = \text{UNDEFINED}$

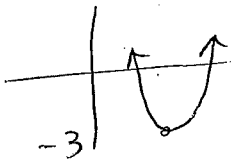
$D = \{x \in \mathbb{R} \mid x \neq 0\}$

$R = \{y \in \mathbb{R} \mid y \neq 0\}$

d) $y = (x-4)^2 - 3$

$D = \{x \in \mathbb{R}\}$

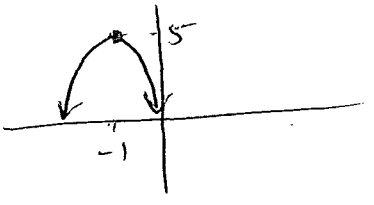
$R = \{y \in \mathbb{R} \mid y \geq -3\}$



e) $y = -2(x+1)^2 + 5$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \leq 5\}$

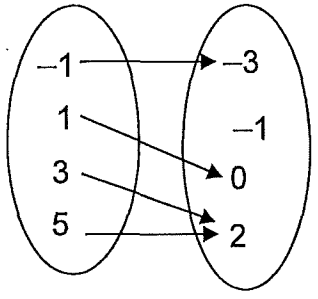


Complete the 1.2 and 1.4 Handout.

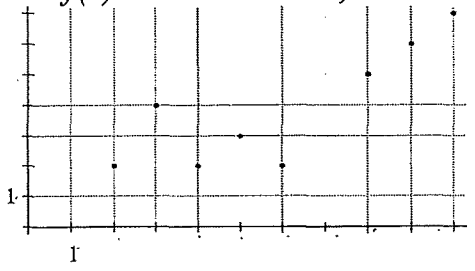
1.2 Function Notation

1. For $f(x) = \{ (-5, 1), (-3, 2) (-1, 3), (1, 2) \}$, determine: a) f(–3) = 2 b) f(1) = 2

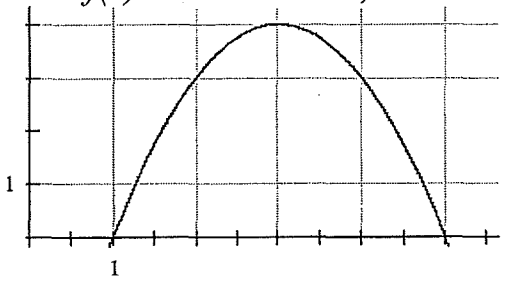
2. For $f(x)$ defined below, determine: a) f(1) = 0 b) f(3) = 2



3. For $f(x)$ defined below, determine: a) f(3) = 4 b) f(8) = 5



4. For $f(x)$ defined below, determine: a) f(2) = 3 b) f(5) = 0



5. For $f(x) = 4x - 5$, determine: a) f(1) = –1 b) f(–2) = $4(-2)-5$ = –13

6. Consider the function $h(n) = 3n^2 + 4n - 8$. Determine

a) h(0) = –8 b) h(1) = –1 c) h(–2) = $3(-2)^2+4(-2)-8$ = $12-8-8=-4$

d) h(1) – h(0) = –1 – (–8) = 7

e) h(2a) = $3(2a)^2+4(2a)-8$ = $12a^2+8a-8$

f) h(a+1) – h(a)

$= 3(a+1)^2+4(a+1)-8-[3a^2+4a-8]$

$= 3(a^2+2a+1)+4a+4-8-3a^2-4a+8$

$= 3a^2+6a+3+4a+4-8-3a^2-4a+8$

$= 6a+7$

7. For $g(x) = 6 - 4x$, determine values of x for which a) $g(x) = 2$ b) $g(x) = 14$

a) $6-4x=2$

$-4x=2-6$

$-4x=-4$

b) $6-4x=14$

$-4x=8$

$x=-2$

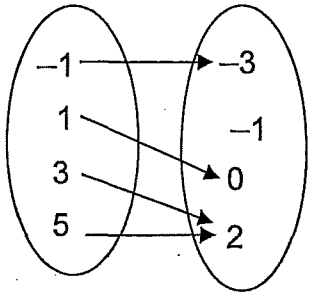
Day 3 homework

1.4 Domain and Range of a Function

8. For $f(x) = \{ (-5, 1), (-3, 2) (-1, 3), (1, 2) \}$, determine the domain and range.

$D = \{-5, -3, -1, 1\}$ $R = \{1, 2, 3\}$

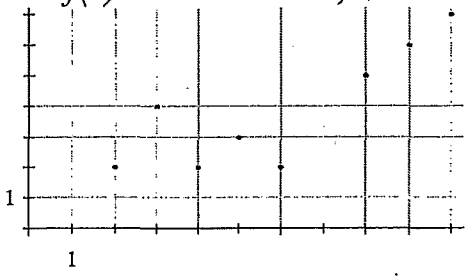
9. For $f(x)$ defined below, determine the domain and range.



$D = \{-1, 1, 3, 5\}$

$R = \{-3, 0, 2\}$

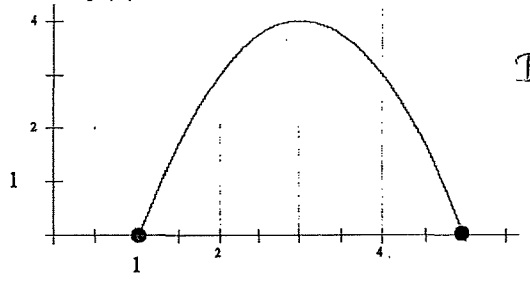
10. For $f(x)$ defined below, determine the domain and range.



$D = \{2, 3, 4, 5, 6, 8, 9, 10\}$

$R = \{2, 3, 4, 5, 6, 7\}$

11. For $f(x)$ defined below, determine the domain and range.



$D = \{x \in \mathbb{R} \mid -1 \leq x \leq 5\}$

$R = \{y \in \mathbb{R} \mid 0 \leq y \leq 4\}$

12. For $f(x) = 4x - 5$, determine the domain and range $D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R}\}$

13. Consider the function $h(n) = -3(n-5)^2 + 8$. Determine the domain and range.

$D = \{n \in \mathbb{R}\}$

$R = \{h(n) \in \mathbb{R} \mid h(n) \leq 8\}$

Solutions

1. a) 2 b) 2 2. a) 0 b) 2 3. a) 4 b) 5 4. a) 3 b) 0 5. a) −1 b) −13
6. a) −8 b) −1 c) −4 d) 7 e) $12a^2 + 8a - 8$ f) $6a + 7$ 7. a) 1 b) −2

8. $D = \{-5, -3, -1, 1\}$ $R = \{1, 2, 3\}$ 9. $D = \{-1, 1, 3, 5\}$ $R = \{-3, 0, 2\}$
10. $D = \{2, 3, 4, 5, 6, 8, 9, 10\}$ $R = \{2, 3, 4, 5, 6, 7\}$ or $R = \{y \in I \mid 2 \leq y \leq 7\}$
11. $D = \{x \in \Re \mid 1 \leq x \leq 5\}$ $R = \{y \in \Re \mid 0 \leq y \leq 4\}$
12. $D = \{x \in \Re\}$ $R = \{y \in \Re\}$ 13. $D = \{n \in \Re\}$ $R = \{h(n) \in \Re / h(n) \leq 8\}$

1.1 Relations and Functions

1. State which relations are functions. Explain.

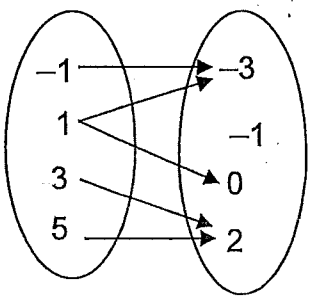
a) { (–5, 1), (–3, 2) (–1, 3), (1,2) }

Yes. x not repeated.

b) { (0, 4), (3, 5) (5, –2), (0, 1) }

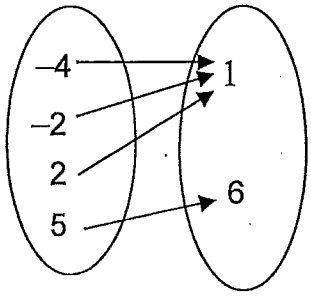
No. $x=0$ has 2 y-values.

c)



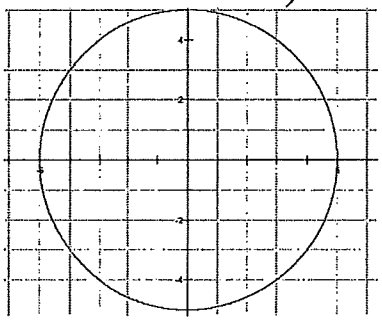
No. $x=1$ has 2 y-values.

d)



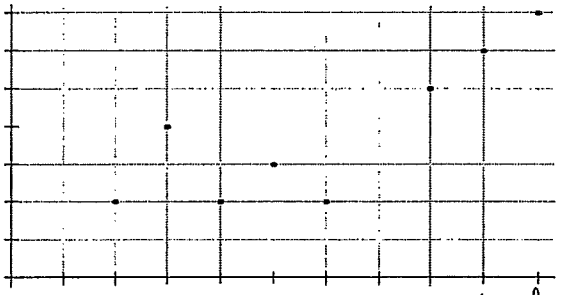
Yes. x not repeated.

e)



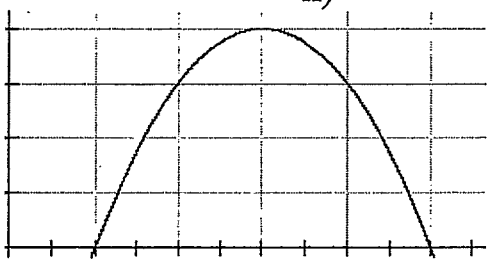
No. fails VLT

f)



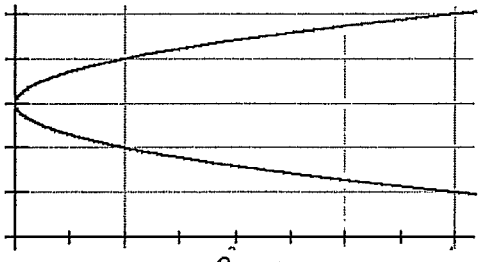
Yes. x not repeated.

g)



Yes. passes VLT

h)



No. fails VLT.

i) $y=3-x$

Yes Linear

Passes VLT.

j) $y=3(x+4)^2-1$

Yes. parabola

Passes VLT

k) $x^2+y^2=4$

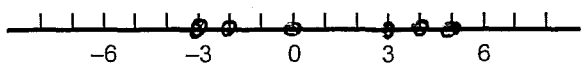
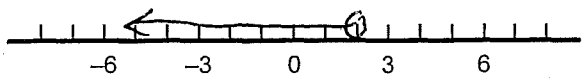
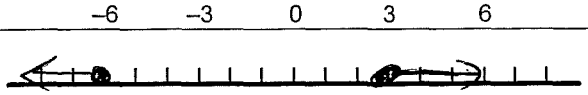
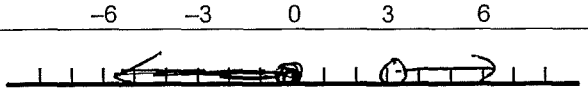
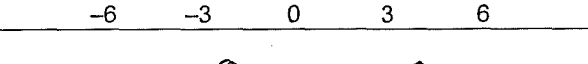
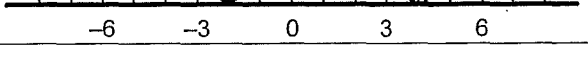
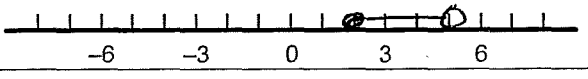
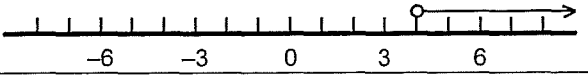
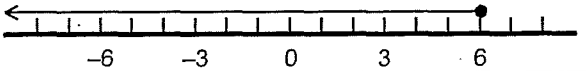
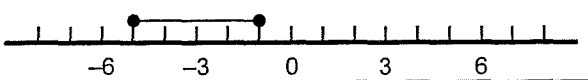
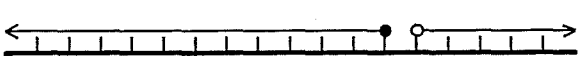
NO. Circle.

fails VLT

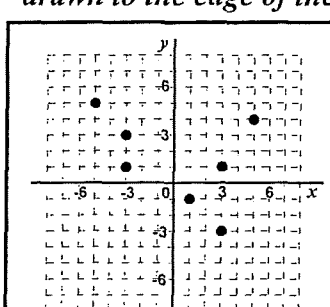
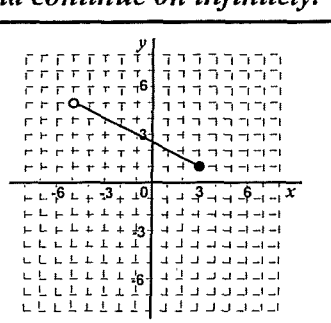
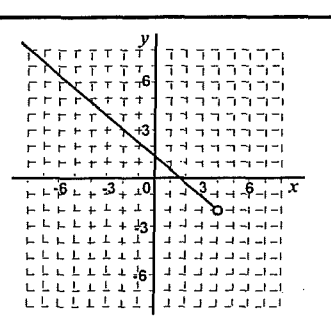
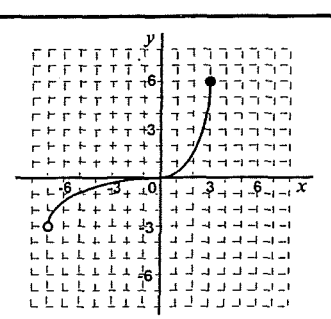
Solutions
Functions: a, d, f, g, i, j

Domain & Range Date:

1. Complete the table:

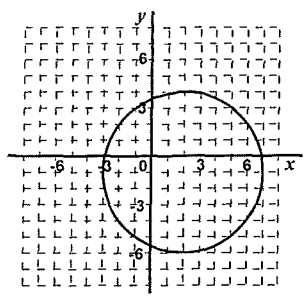
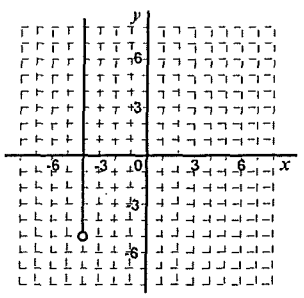
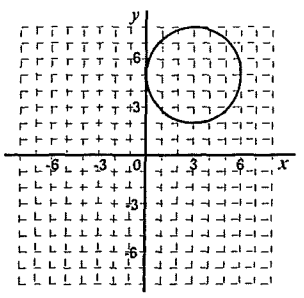
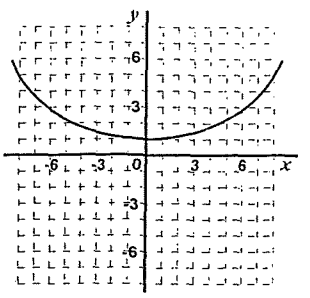
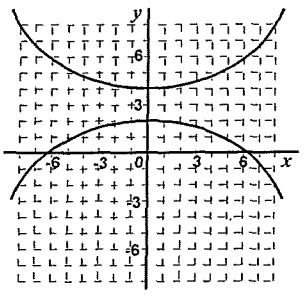
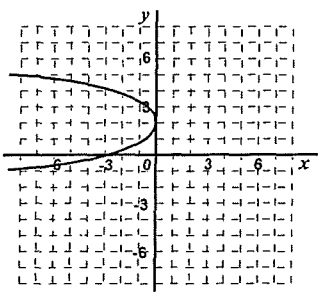
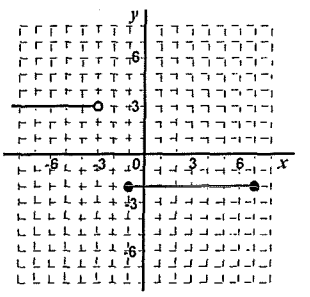
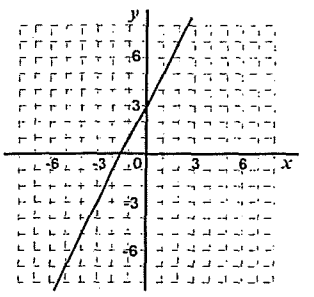
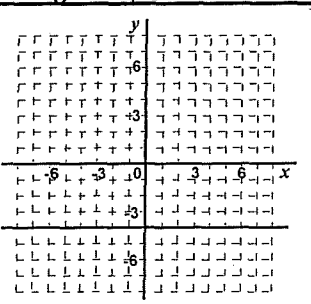
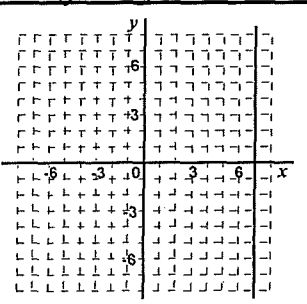
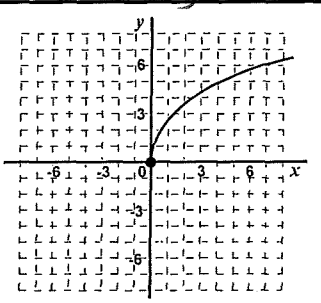
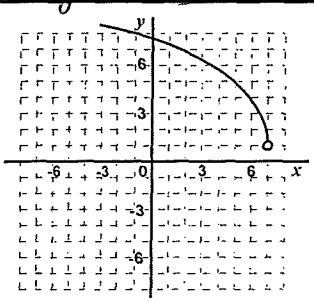
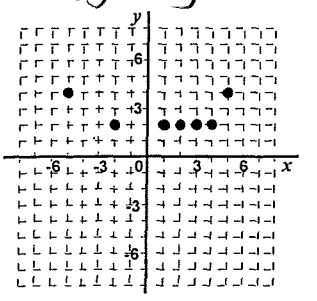
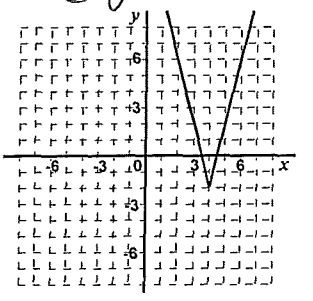
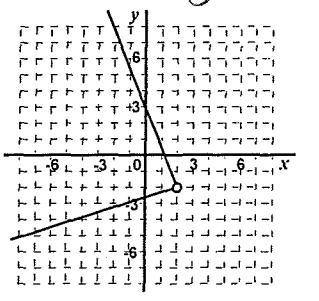
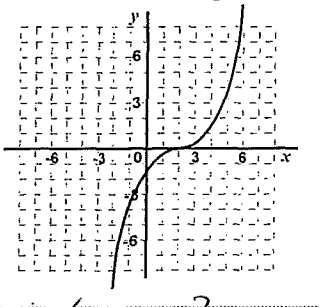
#	Graph	Expression
1.		$\{-3,-2,0,3,4,5\}$
2.		$x < -4$
3.		$x \geq -1$
4.		$x \leq -6$ or $x \geq 3$
5.		$x \leq 0$ or $x > 3$
6.		$-2 < x < 4$
7.		$2 \leq x < 5$
8.		$x > 4$
9.		$x \leq 6$
10.		$-5 \leq x \leq -1$
11.		$x \leq 3$ or $x > 4$
12.		$x \in \mathbb{R}$

State the Domain and Range of each of the given relations in the space provided. ***Assume that graphs drawn to the edge of the grid continue on infinitely.***

			
D: $\{-5,-3,1,3,5\}$	*D:* $\{x \in \mathbb{R} \mid -5 < x \leq 3\}$	*D:* $\{x \in \mathbb{R} \mid x < 4\}$	*D:* $\{x \in \mathbb{R} \mid -7 \leq x \leq 3\}$
R: $\{-3,-1,1,3,4,5\}$	*R:* $\{y \in \mathbb{R} \mid 1 \leq y < 5\}$	*R:* $\{y \in \mathbb{R} \mid y > -2\}$	*R:* $\{y \in \mathbb{R} \mid -3 < y \leq 6\}$

Domain & Range

Date:

			
D: $\{x \in \mathbb{R} \mid -3 \le x \le 7\}$	D: $\{-4\}$	D: $\{x \in \mathbb{R} \mid 0 \le x \le 6\}$	D: $\{x \in \mathbb{R}\}$
R: $\{y \in \mathbb{R} \mid -6 \le y \le 4$	R: $\{y \in \mathbb{R} \mid y > -5\}$	R: $\{y \in \mathbb{R} \mid 2 \le y \le 8\}$	R: $\{y \in \mathbb{R} \mid y \ge 1\}$
			
D: $\{x \in \mathbb{R}\}$	D: $\{x \in \mathbb{R} \mid x \le 0\}$	D: $\{x \in \mathbb{R} \mid x < -3, -1 \le x \le 7\}$	D: $\{x \in \mathbb{R}\}$
R: $\{y \in \mathbb{R} \mid y \le 2$ or $y > 4\}$	R: $\{y \in \mathbb{R}\}$	R: $\{-2, 3\}$	R: $\{y \in \mathbb{R}\}$
			
D: $\{x \in \mathbb{R}\}$	D: $\{7\}$	D: $\{x \in \mathbb{R} \mid x \ge 0\}$	D: $\{x \in \mathbb{R} \mid x < 7\}$
R: $\{y = 4\}$	R: $\{y \in \mathbb{R}\}$	R: $\{y \in \mathbb{R} \mid y \ge 0\}$	R: $\{y \in \mathbb{R} \mid y > 1\}$
			
D: $\{-5, -2, 1, 2, 3, 4, 5\}$	D: $\{x \in \mathbb{R}\}$	D: $\{x \in \mathbb{R} \mid x < 2\}$	D: $\{x \in \mathbb{R}\}$
R: $\{2, 4\}$	R: $\{y \in \mathbb{R} \mid y \ge -2\}$	R: $\{y \in \mathbb{R} \mid y < -2$ or $y > -2\}$	R: $\{y \in \mathbb{R}\}$