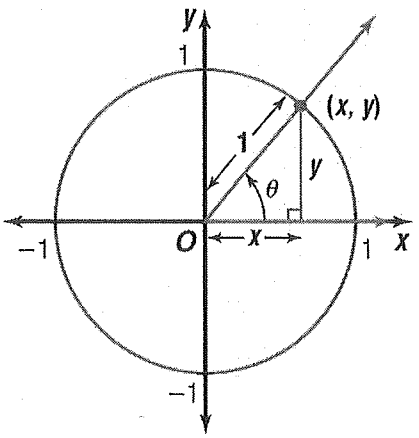
Day 3 – Unit Circle & Trigonometric Ratios

A circle with **radius 1** is called a **unit circle**. A unit circle is tool that can be used to find the exact value of a trig ratio **without a calculator**.



The trig ratio for any angle θ can be found using the following relationships:

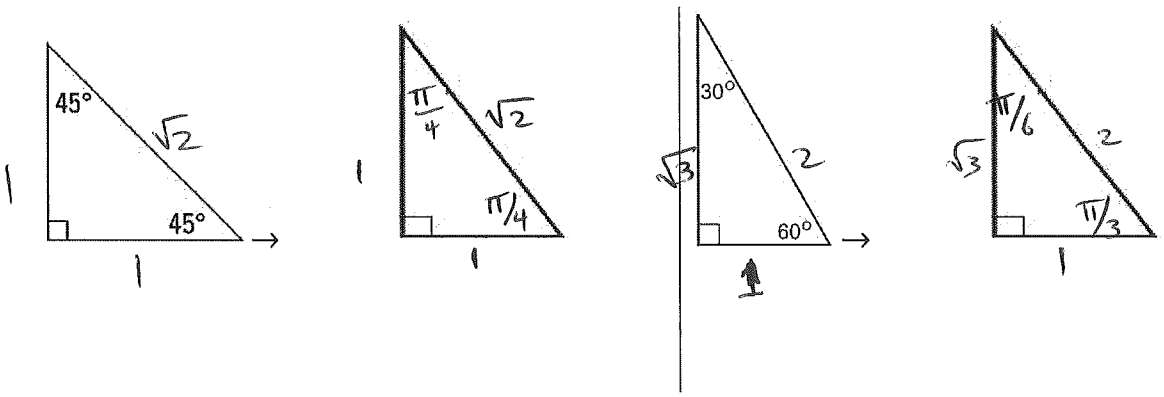
$\sin\theta = \frac{y}{1} = y$

$\cos\theta = \frac{x}{1} = x$

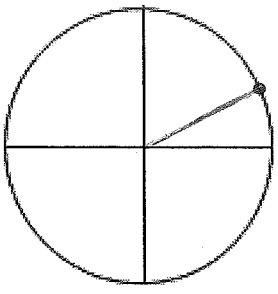
$\tan\theta = \frac{y}{x}$

We will use the unit circle to find **exact values** of trig ratios for the special angles **30°, 45°, 60°**, and their multiples. We need to rewrite our special triangles with a **hypotenuse of 1, and in radian measure.**

EX 1 -Rewriting our special triangles:



EX 2 - Find the exact value of $\cos\frac{\pi}{6}$ and $\sin\frac{\pi}{6}$



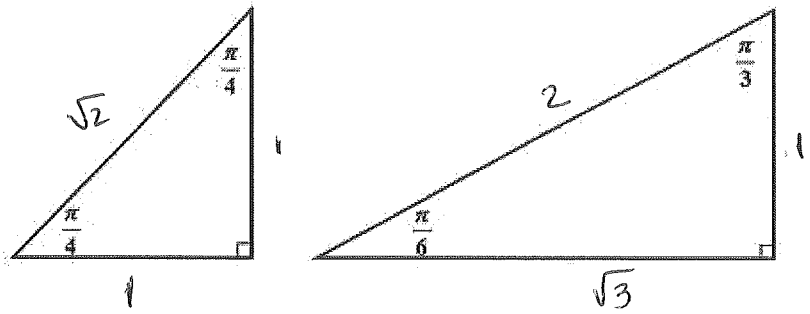
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

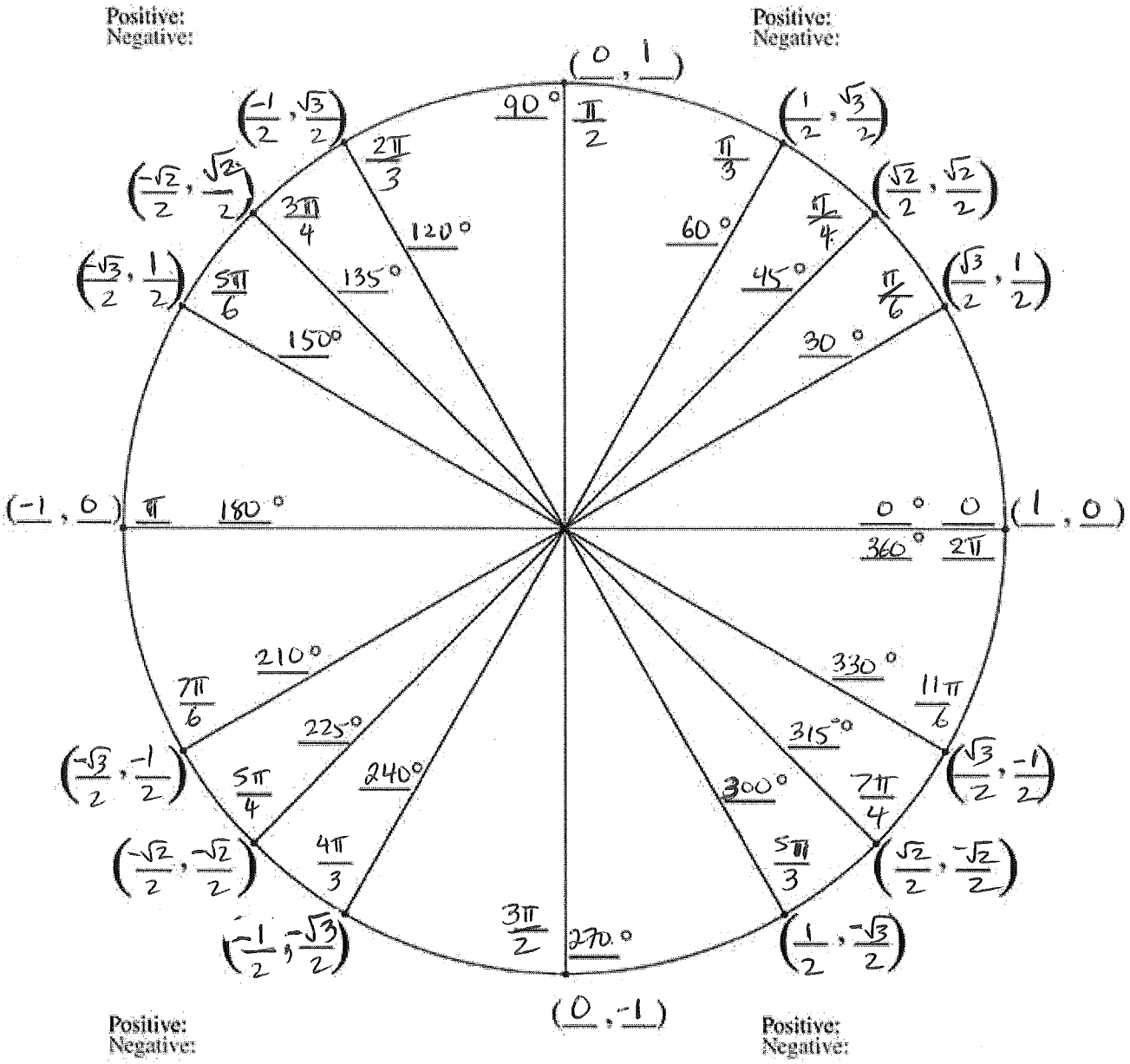
$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

** Now we will build the unit circle to find exact values of multiples of the special angles.*

EX 3 – Build the Unit Circle using: special triangles, trig ratios, CAST





EX 4 - Use the unit circle to determine the exact value of $\sin\frac{5\pi}{3}$. → y coordinate of

$= -\frac{\sqrt{3}}{2}$ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

OR $\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

EX 5 - Use the unit circle to determine the exact values of the (6) trig ratios for angle $\frac{5\pi}{6}$

$\sin\frac{5\pi}{6} = \frac{1}{2}$ $\csc\frac{5\pi}{6} = 2$

$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ $\sec\frac{5\pi}{6} = \frac{-2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ $\cot\frac{5\pi}{6} = -\sqrt{3}$

Unit Circle – More Practice Questions

1. Use the unit circle to determine the exact values of the (6) trig ratios for an angle of $\frac{7\pi}{4}$.

$\sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$ $\csc\frac{7\pi}{4} = -\frac{2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

$\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$ $\sec\frac{7\pi}{4} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\tan\frac{7\pi}{4} = -1$ $\cot\frac{7\pi}{4} = -1$

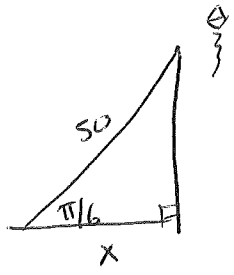
2. Use the unit circle to determine the exact value for $\dfrac{\csc\frac{3\pi}{4}\cos\frac{5\pi}{5}}{\tan\frac{\pi}{6}+\sec\frac{11\pi}{6}}$

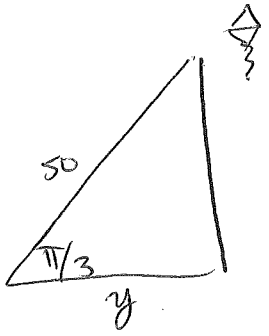
$= \dfrac{\left(\frac{+2}{\sqrt{2}}\right)(-1)}{\frac{1}{\sqrt{3}}+\frac{2}{\sqrt{3}}}$

$= -\dfrac{2}{\sqrt{2}} \div \left(\dfrac{3}{\sqrt{3}}\right)$

$= -\dfrac{2}{\sqrt{2}} \times \dfrac{\sqrt{3}}{3} = -\dfrac{2\sqrt{3}}{3\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{-2\sqrt{6}}{6} = \dfrac{-\sqrt{6}}{3}$

3. Ravinder is flying his kite at the end of a 50 m long string. The string makes an angle of $\frac{\pi}{6}$ with the ground. The wind speed increases and the kite flies higher until the string makes an angle of $\frac{\pi}{3}$ with the ground. Determine an exact expression for the horizontal distance that the shadow of the kite moves between the two positions.





$\cos\frac{\pi}{6} = \frac{x}{50}$

$x = 50\cos\frac{\pi}{6}$

$= (50)\left(\frac{\sqrt{3}}{2}\right)$

$= 25\sqrt{3}$

$\cos\frac{\pi}{3} = \frac{50}{y}$

$y = 50\left(\frac{1}{2}\right)$

$= 25$

$\therefore 25\sqrt{3} - 25 = 25(\sqrt{3}-1)$ m along the horizontal distance.

Day 2-3 Practice - *Without a calculator*

1) A circle has radius 20 cm. Determine the length of the arc subtended by an angle of $\frac{3}{4}$.

$a = r\theta = 20\left(\frac{3}{4}\right) = \frac{60}{4} = 15\text{ cm}$

2) Determine the exact radian measure of 110°.

$= 110\left(\frac{\pi}{180}\right) = \frac{11\pi}{18}$

3) A child rides a carousel that completes 20 revolutions in 2 minutes. Determine the child's angular velocity.

$\omega = \frac{\theta}{t} = \frac{20(2\pi)}{120} = \frac{40\pi}{120} = \frac{\pi}{3}$ rad/s.

OR $\frac{40\pi}{2} = 20\pi$ rad/minute.

4) Determine the exact values of the (6) trigonometric ratios for the angle $\frac{4\pi}{3}$.

$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\quad \csc\frac{4\pi}{3} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cos\left(\frac{4\pi}{3}\right) = \frac{-1}{2}$ $\quad \sec\frac{4\pi}{3} = -2$

$\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$ $\quad \cot\frac{4\pi}{3} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

5) Use the unit circle to determine the exact value for $\sin\left(\frac{5\pi}{4}\right) - \cos\left(\frac{11\pi}{6}\right)\cot\left(\frac{\pi}{3}\right)$

$= -\frac{\sqrt{2}}{2} - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2} \div \frac{\sqrt{3}}{2}\right)$

$= -\frac{\sqrt{2}}{2} - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right)$

$= -\frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{-(\sqrt{2}+1)}{2}$