

$$\begin{aligned} \text{a. } & \sqrt{\sqrt{x^{12}}} \\ &= \left((x^{12})^{\frac{1}{2}} \right)^{\frac{1}{2}} \\ &= (x^6)^{\frac{1}{2}} \\ &= x^3 \end{aligned}$$

$$\begin{aligned} \text{b. } & \sqrt[3]{\sqrt{y^4}} \\ &= \left((y^4)^{\frac{1}{3}} \right)^{\frac{1}{2}} \\ &= (y^{4/3})^{\frac{1}{2}} \\ &= y^{\frac{4}{6}} = y^{\frac{2}{3}} \end{aligned}$$

$$\begin{aligned} \text{c. } & \sqrt{81m^8} = (81)^{\frac{1}{2}} (m^8)^{\frac{1}{2}} \\ &= 9m^4 \end{aligned}$$

$$\begin{aligned} \text{d. } & \sqrt{\sqrt{10000y^6}} \\ &= \left((10000y^6)^{\frac{1}{2}} \right)^{\frac{1}{2}} \\ &= 10000^{\frac{1}{4}} y^{\frac{3}{2}} \\ &= 10y^{\frac{3}{2}} \end{aligned}$$

$$\begin{aligned} \text{e. } & \sqrt[3]{\sqrt{64x^{18}}} \\ &= \left((64x^{18})^{\frac{1}{3}} \right)^{\frac{1}{2}} \\ &= (4x^6)^{\frac{1}{2}} \\ &= 2x^3 \end{aligned}$$

$$\begin{aligned} \text{f. } & \sqrt[5]{\frac{\sqrt{x}\sqrt{x^3}}{x^{\frac{3}{4}}}} \\ &= \left(\frac{x^{\frac{1}{2}} x^{\frac{3}{2}}}{x^{\frac{3}{4}}} \right)^{\frac{1}{5}} \\ &= \left(x^{\frac{1}{2} + \frac{3}{2} - \frac{3}{4}} \right)^{\frac{1}{5}} \\ &= \left(x^{\frac{4}{2} - \frac{3}{4}} \right)^{\frac{1}{5}} = \left(x^{\frac{5}{4}} \right)^{\frac{1}{5}} \\ &= x^{\frac{1}{4}} \end{aligned}$$

$$\begin{aligned} \text{g. } & \left(\frac{-8x^3}{216} \right)^{\frac{1}{3}} \\ &= \left(\frac{216}{-8x^3} \right)^{\frac{1}{3}} \\ &= \sqrt[3]{\frac{216}{-8x^3}} = \frac{6}{-2x} = \frac{-3}{x} \end{aligned}$$

$$\begin{aligned} \text{h. } & \left(\frac{16}{81y^8} \right)^{\frac{3}{4}} \\ &= \left(\frac{4\sqrt[4]{16}}{\sqrt[4]{81y^8}} \right)^3 = \left(\frac{2}{3y^2} \right)^3 \\ &= \frac{8}{27y^6} \end{aligned}$$

$$\begin{aligned} \text{i. } & \frac{(2x^{-3}y^2)^3}{(x^3y^{-4})^2} \\ &= \frac{2^3 x^{-9} y^6}{x^6 y^{-8}} \\ &= 8x^{-15} y^{14} = \frac{8y^{14}}{x^{15}} \end{aligned}$$

$$\begin{aligned} \text{j. } & \frac{(27a^{-3}b^{12})^{\frac{1}{3}}}{(16a^{-8}b^{12})^{\frac{1}{2}}} \\ &= \frac{3a^{-1}b^4}{4a^{-4}b^6} \\ &= \frac{3a^3}{4b^2} \end{aligned}$$

Practice:1. Rational Exponents of the form $\frac{m}{n}$

a. $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

b. $81^{\frac{5}{4}} = (\sqrt[4]{81})^5 = 3^5 = 243$

c. $\left(\frac{49}{81}\right)^{-\frac{3}{2}} = \left(\frac{81}{49}\right)^{\frac{3}{2}} = \left(\frac{9}{7}\right)^3 = \frac{729}{343}$

2. Evaluate rational exponents of form $\frac{1}{n}$

a. $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{16}{81}} = \frac{2}{3}$

b. $(-32)^{\frac{1}{5}} = \sqrt[5]{-32} = -2$

c. $(-16)^{\frac{1}{4}} = \sqrt[4]{-16}$ NOT defined/possible

d. $(-27)^{-\frac{1}{3}} = \frac{1}{(-27)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{-27}} = -\frac{1}{3}$

OR $-\frac{1}{3}$ OR $-\frac{1}{3}$

3. Simplify. Express your answer using only positive exponents.

a. $\frac{x^{\frac{2}{3}}x^{\frac{2}{3}}}{x^{\frac{1}{3}}} = \frac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}} = x^{\frac{4}{3}-\frac{1}{3}} = x^1 = x$

b. $(5x^{\frac{1}{2}})^2 (4x^{-\frac{1}{2}})$

$$= (25x)(4x^{-\frac{1}{2}})$$

$$= 100x^{\frac{1}{2}}$$

Day 3: Simplifying Rational Exponents

Chapter 4: Exponential Functions

4. Express $\sqrt{5}$ as an expression with exponents. = $5^{\frac{1}{2}}$

5. Express $\sqrt[6]{7}$ as an expression with exponents. = $7^{\frac{1}{6}}$

$$7^{\frac{5}{6}}$$

6. Express $\sqrt[4]{10000m^8}$ as an expression with exponents, then evaluate.

$$= (10000)^{\frac{1}{4}} (m^8)^{\frac{1}{4}}$$

$$= 10 m^2$$

7. Is it easier to calculate $16^{\frac{3}{2}}$ by cubing then square rooting, or by square rooting then cubing?

Square rooting first then cubing it.

8. Simplify.

a) $(n^4)^{\frac{3}{2}}$

$$= n^{4(\frac{3}{2})}$$

$$= n^6$$

b) $(27p^6)^{\frac{2}{3}}$

$$= (\sqrt[3]{27p^6})^2$$

$$= (3p^2)^2 = 9p^4$$

c) $(25b^6)^{-1.5} = (25b^6)^{-\frac{3}{2}}$

$$= \frac{1}{(25b^6)^{\frac{3}{2}}} = \frac{1}{(5b^3)^3}$$

$$= \frac{1}{125b^9}$$

d) $(9r^4)^{0.5}$

$$= (9r^4)^{\frac{1}{2}}$$

$$= 3r^2$$

e) $(81x^{12})^{1.25}$

$$= (81x^{12})^{\frac{5}{4}}$$

$$= (3x^3)^5$$

$$= 243x^{15}$$

f) $(5p^{\frac{3}{2}})^{-2}$

$$= \frac{1}{(5p^{\frac{3}{2}})^2} = \frac{1}{25p^3}$$

g) $(3b^{\frac{1}{2}})(6b^{\frac{4}{3}})$

$$= 18 b^{\frac{1}{2} + \frac{4}{3}}$$

$$= 18 b^{\frac{3}{6} + \frac{8}{6}}$$

$$= 18 b^{\frac{11}{6}}$$

h) $(9a^{\frac{1}{2}})^{\frac{3}{2}}$

$$= 9^{\frac{3}{2}} a^{\frac{3}{4}}$$

$$= (\sqrt{9})^3 a^{\frac{3}{4}}$$

$$= 27 a^{\frac{3}{4}}$$

i) $\frac{4x^2}{2x^{\frac{1}{2}}}$

$$= 2x^{2 - \frac{1}{2}}$$

$$= 2x^{\frac{3}{2}}$$

$$j) \frac{2x^{-\frac{7}{4}}}{(8x)^{\frac{4}{3}}}$$

$$= 2x^{-\frac{7}{4}}$$

$$= \frac{(8^{\frac{4}{3}}) x^{4/3}}{16}$$

$$= \frac{2}{16} x^{-\frac{7}{4} - \frac{4}{3}}$$

$$\frac{1}{8} x^{-\frac{37}{12}} = \frac{1}{8x^{\frac{37}{12}}}$$

$$m) \frac{(\sqrt{9x^3y^2})^3}{\sqrt[4]{16x^{-1}y^{-\frac{2}{3}}}}$$

$$= \frac{(3x^{\frac{3}{2}}y)^3}{(2x^{-\frac{1}{4}}y^{-\frac{2}{12}})}$$

$$= \frac{27x^{\frac{9}{2}}y^3}{2x^{-\frac{1}{4}}y^{-\frac{1}{6}}}$$

$$= \frac{27x^{\frac{9}{2} + \frac{1}{4}}y^{3 + \frac{1}{6}}}{2}$$

$$= \frac{27x^{\frac{19}{4}}y^{\frac{19}{6}}}{2}$$

$$k) \left(\left((a^{-1})(\sqrt[3]{b}) \right) \left(a^{\frac{4}{3}}b^2 \right) \right)^2$$

$$= \left(a^{-1}b^{\frac{1}{3}}a^{\frac{4}{3}}b^2 \right)^2$$

$$= \left(a^{-\frac{7}{3}}b^{\frac{7}{3}} \right)^2$$

$$= a^{-\frac{14}{3}}b^{\frac{14}{3}}$$

$$= \frac{b^{14/3}}{a^{14/3}}$$

$$l) \left(\frac{x^{\frac{1}{2}}y^{-2}}{yx^{-\frac{7}{4}}} \right)^4$$

$$= \left(x^{\frac{1}{2} + \frac{7}{4}}y^{-2-1} \right)^4$$

$$= \left(x^{\frac{9}{4}}y^{-3} \right)^4$$

$$= x^9y^{-12}$$

$$= \frac{x^9}{y^{12}}$$

$$n) (2x^2) \left((64x)^{\frac{1}{3}} \right)^{\frac{1}{2}}$$

$$= 2x^2 \left(4x^{\frac{1}{3}} \right)^{\frac{1}{2}}$$

$$= 2x^2 \left(2x^{\frac{1}{6}} \right)$$

$$= 4x^{2 + \frac{1}{6}}$$

$$= 4x^{\frac{13}{6}}$$

$$o) \frac{3x}{\sqrt[3]{27xy}}$$

$$= \frac{3x}{3x^{\frac{1}{3}}y^{\frac{1}{3}}}$$

$$= \frac{x^{\frac{2}{3}}}{y^{\frac{1}{3}}}$$