Day 3: 6.2 – Introduction to Logarithms

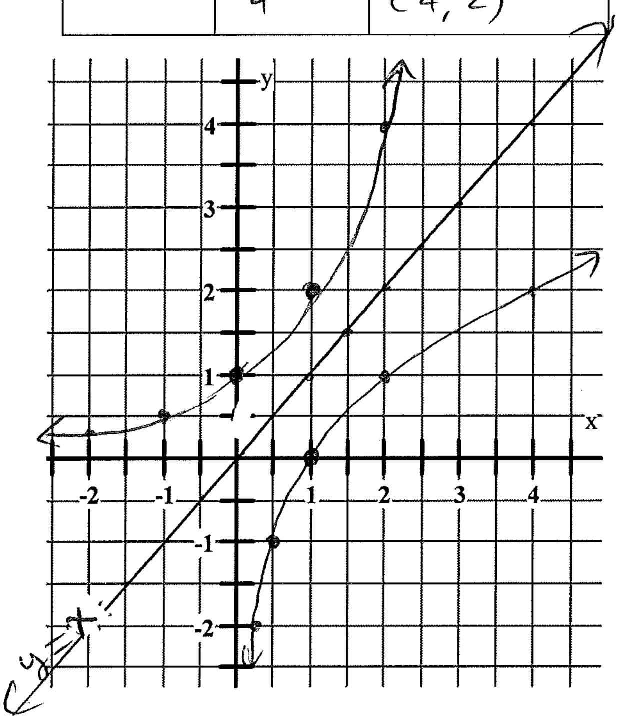
Warm up – Solve the exponential expression: $8^{2x-1} = 4^{-x-6}$

Recall: Inverse of a Function

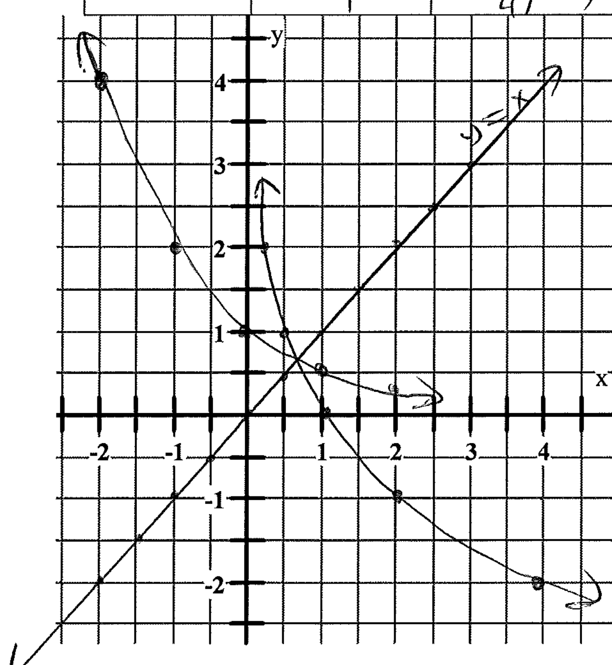
- The inverse function f^{-1} of a function f, is found by writing the function in the form $y = f(x)$, exchanging the values of x and y, and then solving for y
- If you are given co-ordinates of a function f, the co-ordinates of the inverse function f^{-1} can be found by **interchanging x and y.**

EX 1 – a. Graph each exponential function and its' inverse on the same grid.

x	$y = 2^x$	Coordinates of Inverse
-2	$\frac{1}{4}$	$(\frac{1}{4}, -2)$
-1	$\frac{1}{2}$	$(\frac{1}{2}, -1)$
0	1	$(1, 0)$
1	2	$(2, 1)$
2	4	$(4, 2)$



x	$y = \left(\frac{1}{2}\right)^x$	Coordinates of Inverse
-2	4	$(4, -2)$
-1	2	$(2, -1)$
0	1	$(1, 0)$
1	$1/2$	$(1/2, 1)$
2	$1/4$	$(\frac{1}{4}, 2)$



b. **Graph the line $y = x$ on both grids. What do you notice?**

$f(x)$ and $f^{-1}(x)$ are reflected in $y=x$ line.

c. Write the equation for each **inverse** function: $y = 2^x \rightarrow$ $x = 2^y$

$y = \left(\frac{1}{2}\right)^x \rightarrow$ $x = \left(\frac{1}{2}\right)^y$

d. Fill in the following key features of the above graphs:

	$y = 2^x$	$x = 2^y$	$y = \left(\frac{1}{2}\right)^x$	$x = \left(\frac{1}{2}\right)^x$
Domain	$\{x \in R\}$	$\{x \in R \mid x > 0\}$	$\{x \in R\}$	$\{x \in R \mid x > 0\}$
Range	$\{y \in R \mid y > 0\}$	$\{y \in R\}$	$\{y \in R \mid y > 0\}$	$\{y \in \mathbb{R}\}$
x-intercept	none	1	none	1
y-intercept	1	none	1	none
Asymptote(s)	$y = 0$	$x = 0$	$y = 0$	$x = 0$
increasing or decreasing on its domain	Increasing	increasing	decreasing	decreasing

The **inverse** of an exponential function is called a **logarithmic function.**

$x = 2^y$

The **logarithmic function** is defined by $y = \log_b x$, where $b > 0, b \neq 1$

$\log_2 x = y$

- Read as "y equals to the logarithm of x to the base b"
- Any exponential relationship can be written using logarithm notation

A *logarithm* is the **power** to which a number **must be raised** in order to get some **other number.**

If $x = a^y$ then $y = \log_a(x)$

Value = Base $^{\text{Exponent}}$ $\leftrightarrow$ Exponent = log $_{\text{base}}$ (Value)

EX 2 - Rewrite in logarithmic form

a) $5^2 = 25$

$\log_5 25 = 2$

b) $512^{\frac{1}{3}} = 8$

$\log_{512} 8 = \frac{1}{3}$

EX 3 – Rewrite in exponential form

a) $\log_6 36 = 2$

$6^2 = 36$

b) $\log_9 1 = 0$

$9^0 = 1$

Common Logarithms

Logarithms to the base 10 are called **common logarithms.**

- When writing a common logarithm, it is not necessary to write the base; that is; log 100 is understand to mean the same as $log_{10}100$

EX 4 – Evaluate a logarithm

a) $\log_3(81) = x$

$3^x = 81$

$x = 4$

b) $\log 100 = x$

$10^x = 100$

$x = 2$

c) $\log_2(-4) = x$

$2^x = -4$

NOT POSSIBLE

d) $\log(0.01) = x$

$10^x = \frac{1}{100}$

$x = -2$

e) $\log_2\left(\frac{1}{8}\right) = x$

$2^x = \frac{1}{8}$

$\boxed{x = -3}$

f) $\log_2(10) = x$

$2^x = 10$

$x \doteq 3.32$ [Calculator]