Day3-MCR3U

The Amount of an Annuity (FV)

Goal: ***Calculate the amount (future value) of a simple ordinary annuity***
Calculate the present value of a simple ordinary annuity

- An ***annuity*** is a series of ***equal*** payments paid ***in to our out of*** an account at ***regular*** intervals
- In an ***ordinary simple annuity***, payments are made at the ***end*** of each ***compounding*** period
- The ***AMOUNT*** of an annuity (***future value***) is the sum of regular deposits plus ***interest***

Compound Periods *(# times per year)*			
Annually:	1	Semi-Annually:	2
Monthly:	12	Semi-Monthly:	24
Weekly:	52	Bi-weekly:	26
Quarterly:	4	Daily:	365

i = compounding periods per year
n = compounding periods per year

The ***AMOUNT*** of an ordinary simple annuity is given by the formula $A = \dfrac{R\left[(1+i)^n - 1\right]}{i}$, where

A = Future value / amount
i = interest rate per compounding period.
R = deposit / withdrawal per compounding period
n = # of compounding periods

This formula can only be used when the ***payment interval is the same as the compounding period***

Example Suppose $450 were deposited at the end of each quarter for 1.5 years into an annuity that earns 10% per year compounded quarterly

a) What is the amount of the annuity?

$A = ?$

$R = 450$

$i = \dfrac{0.10}{4} = 0.025$

$n = (1.5)(4)$

$= 6$

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 450\left[\frac{1.025^6 - 1}{0.025}\right]$$

$$= \$2874.48$$

The ***INTEREST*** of an ordinary simple annuity is given by the formula $I = A - Rn$, where I is interest amount

b) How much interest did the annuity earn?

$I = 2874.48 - 450(6)$

$= \$174.48$

Example 1: R

Jane invests $500 at the end of each year for 4 years. If interest is paid at 7%/a compounded annually, how much will the investment be worth immediately after the last deposit is made? What is the total interest earned?

$A = ?$

$R = 500$

$i = 0.07$

$n = 4$

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

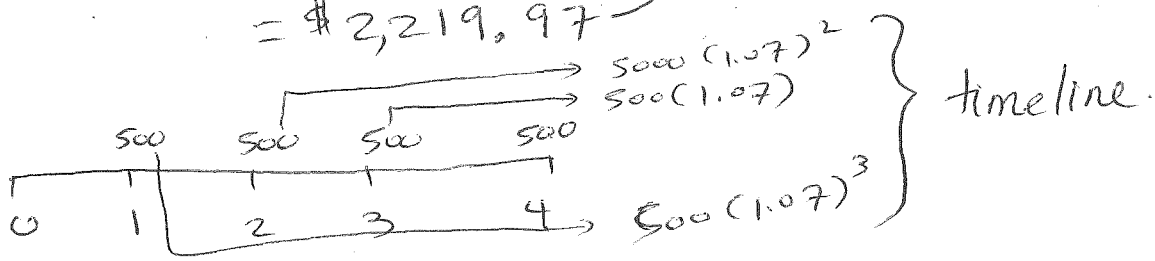
$$= 500\left(\frac{1.07^4 - 1}{0.07}\right)$$

$$= \$2,219.97$$

Interest:

$2219.97 - 4(500)$

$= \$219.97$



Example 2:

Determine the future value of quarterly deposits of $1000 over 10 years at 5½%/a compounded quarterly.

$i = \frac{0.055}{4}$

$$A = 1000\left[\frac{(1.01375)^{40} - 1}{0.01375}\right]$$

$$= 52\,856.06$$

Example 3:

How much must be deposited each month at 6%/a compounded monthly for 3 years in order to have $10000 on the day of the last deposit?

$A = 10\,000$

$i = \frac{0.06}{12} = 0.005$

$n = (12)(3) = 36$

$R = ?$

$$10000 = R\left[\frac{(1+0.005)^{36} - 1}{0.005}\right]$$

$$10000 = R[39.336]$$

$$R = 254.22$$

Example 4:

a) Laars deposits $2000 each quarter year for 6½ years. If interest is at 5.5% C_q, how much will be in his account after the last deposit is made?

b) How much interest has his money earned in total?

$R = 2000$

$i = \frac{0.055}{4} = 0.01375$

$n = (6.5)(4) = 26$

$A = ?$

$A = 2000\left[\frac{(1+0.01375)^{26}-1}{0.01375}\right]$

$= 62003.29$

$I = A - Rn$

$= 62003.29 - (2000)(26)$

$= 10003.29$

Annuities Practice

1. a) How much must Hallie deposit each month in a savings account paying 3%/a C_m in order to have $20 000 after 5¾ years? $R = 265.93$

   b) How much interest did her investment earn? $I = 2000 - (265.93)(69)$

   $= \$1650.50$

2. Once every 3 months, James deposits $100 into a savings account paying 4%/a C_q. How much will he have after 51 months?

   $A = 1843.04$

3. Aziz invests $1200 each year in an account paying 6%/a C_a while Chee-Mei invests $100 per month into an account paying 5.88%/a C_m. Who has more after 10 years and how much more do they have?

   Aziz: $A = 15,816.95$ Chee-Mei: $A = 16281.65$

   Chee-Mei has more money ($\$464.70$)

4. Harry's parents saved for his college education by depositing $1200 at the end of every 6 months since birth in a *Registered Education Savings Plan* (RESP) that earns 5%/a compounded semi-annually.

   a) What is the amount of the RESP at the end of 18 years? $\$68,761.70$

   b) How much interest is earned? $\$25,561.70$

   c) How much extra interest would have been earned if the interest rate was 7% per year compounded semi-annually?

   $FV = 84,009.12$

   $I = 40,809.12$

   Extra Int $= 40,809.12 - 25,561.70$

   $\$15247.42$

Homework: p. 511 #5 (draw timeline for b), 6-9