Day 2: 6.1 Introduction to Vectors

Classification of Quantities:

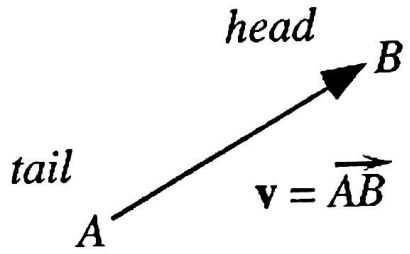
Scalar is a quantity having magnitude (ie, measured numerically)
Example:

Vector is a quantity having *both* magnitude *and* direction
Example:

A vector is represented by an **arrow** with an initial point (tail) and an end point at the arrow (head).

Example: a vector that starts at point A and ends at point B.

The **magnitude** (length) of the vector is **a positive real number**. The arrow indicates the **direction** of the vector.



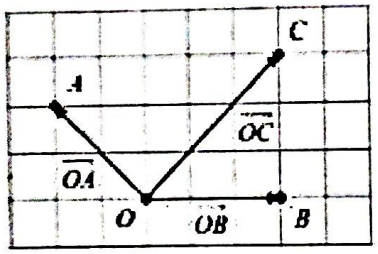
Geometric Vectors are vectors not related to any coordinate system. For example, the *directed line segment* $\overrightarrow{AB}$

Algebraic Vectors are vectors related to a coordinate system. These vectors are (in general) described by their *components* relative to a reference system (frame). For example $v = (2,3,-1)$.

The *position vector* is the directed line segment OP from **the origin** of the coordinate system O to a generic point P.

The *displacement vector* AB is the directed line segment from the point A to the point B .

Example: position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$.

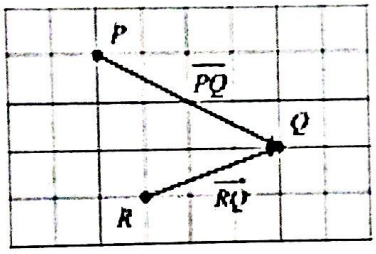


$\overrightarrow{OA} = [-2, 2]$ 2 left 2up

$\overrightarrow{OB} = [3, 0]$ 3 Right

$\overrightarrow{OC} = [2, 3]$ 2R 3up.

displacement vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$.

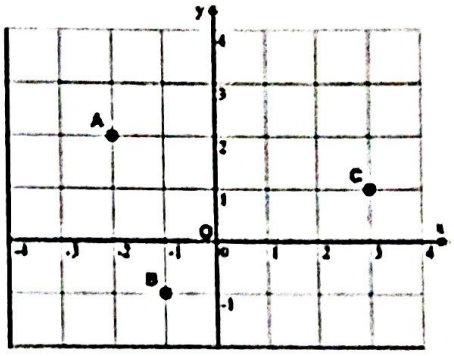


$\overrightarrow{PQ} = [4, -2]$

$\overrightarrow{RQ} = [3, 1]$

The Magnitude is the length, size, norm or intensity of the vector. The magnitude of the vector v is denoted by $|\vec{v}|$ *or* $||\vec{v}||$ *or* v

Find the magnitude of the following vectors:

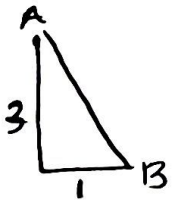


a) $|\overrightarrow{OA}| = \sqrt{2^2+2^2} = \sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

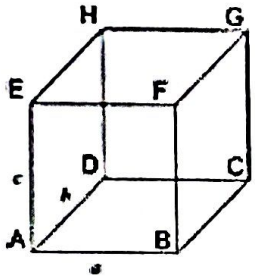
b) $|\overrightarrow{AB}| = \sqrt{3^2+1^2} = \sqrt{10}$

c) $|\overrightarrow{BC}| = \sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$

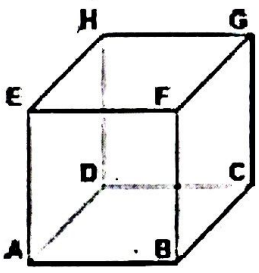
NOTE:



3D Pythagorean Theorem



If AB = 10 cm, find the magnitude of:



a) $|\overrightarrow{AB}| = 10$ cm

b) $|\overrightarrow{BD}| = \sqrt{10^2+10^2} = \sqrt{200} = 10\sqrt{2}$

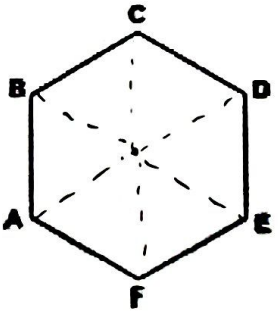
c) $|\overrightarrow{BH}| = \sqrt{10^2+10^2+10^2} = \sqrt{300} = 10\sqrt{3}$

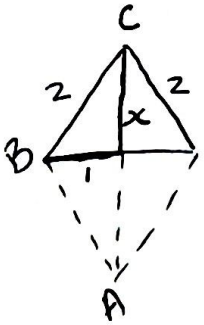
Consider the regular hexagon *ABCDEF* with the side length equal to 2 *m*. Find the magnitude of the following vectors:

a) $\overrightarrow{AB}$ $|\overrightarrow{AB}| = 2$ m

b) $|\overrightarrow{AC}| = 2x = 2\sqrt{3}$

c) $\overrightarrow{AD}$ $|\overrightarrow{AD}| = 4$ m





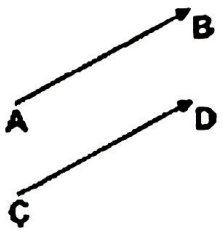
$x^2+1^2 = 4$

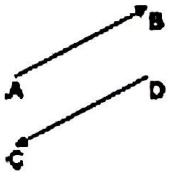
$x^2 = 3$

$x = \sqrt{3}$

Two vectors are equal iff:

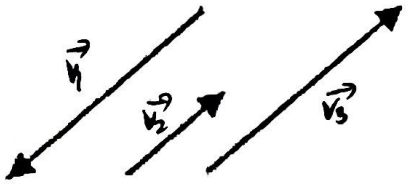
1. They are parallel to each other and their directions are the same.
2. Their magnitudes are the same.



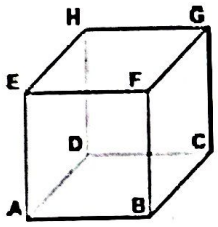


Two vectors are **opposite** if they have the same magnitude but point in opposite directions. When two vectors are opposite, one is the negative of the other.

Two vectors are **parallel** if their directions are the same or opposite.



Find three pairs of equivalent vectors and three pairs of opposite vectors in the diagram:



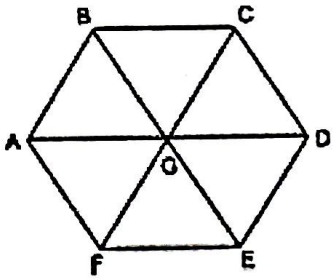
$\vec{AE} = \vec{BF}$

$\vec{AF} = \vec{DG}$

$\vec{EB} = \vec{HC}$.

Example: ABCDEF is a regular hexagon. Give examples of vectors which are:

a) equal $\vec{BC} = \vec{FE}$, $\vec{FG} = \vec{GC}$



b) parallel but having different magnitudes

$\vec{BC}$ and $\vec{AD}$, $\vec{AB}$ and $\vec{FC}$

c) equal in magnitude, but opposite in direction

$\vec{AB}$ and $\vec{EC}$, $\vec{AG}$ and $\vec{DG}$

d) equal in magnitude, but not parallel

$\vec{BG}$ and $\vec{GF}$, $\vec{BA}$ and $\vec{AF}$

e) different in both magnitude and direction

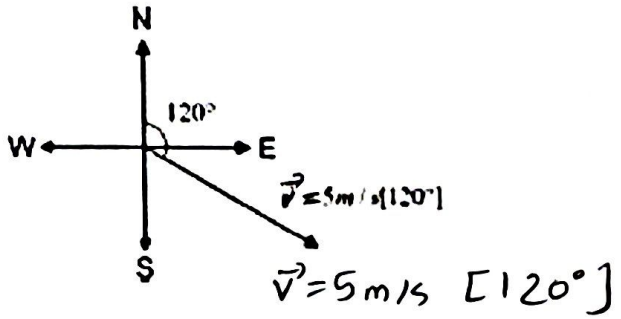
$\vec{FE}$ and $\vec{FC}$

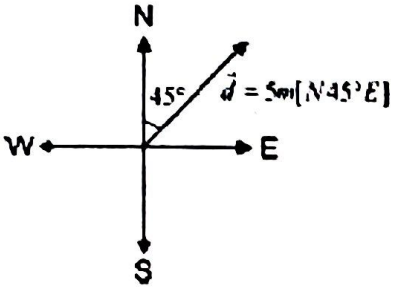
$\vec{AB}$ and $\vec{AD}$

NOTE: These are some possible answers. There are more possibilities

Direction:
To express the direction of a vector in a horizontal plane, the following standards are used. Note. Because we use a reference system, the following vectors may be considered also algebraic.

True Bearing The direction of the vector is given by the angle **between the North and the vector, measured in a clockwise direction.**
Example: $v=5m/s$ [120°].

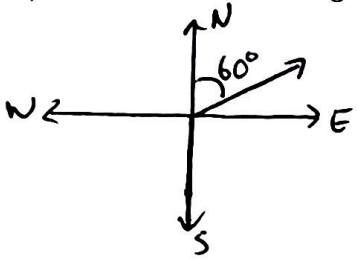




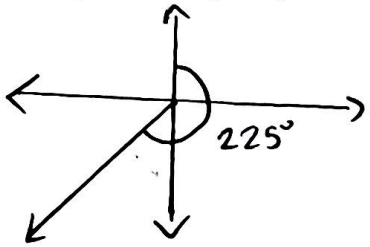
Quadrant Bearing The direction is given by the angle between the North-South line and the vector.
Example: $5m[N45°E]$. Read: 45° East of North.
Note. $5m[N45°E]=5m[NE]$ Read: $5m$ North-East.

Ex 10. Draw each vector given by magnitude and true bearing.

a) $r = 2m$ at a true bearing of [60°]

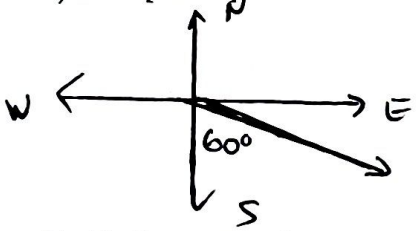


b) $a = 5m/s$ [225°]

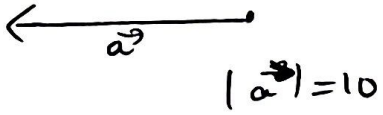


Ex 11. Draw each vectors given by magnitude and quadrant bearing.

a) $d=2m[S60°E]$

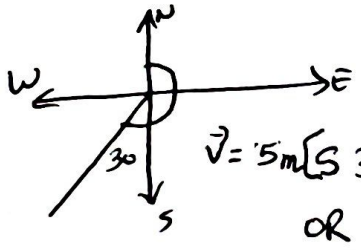


b) $F=10N[W]$



Ex 12. Convert each vector.

a) $v = 5m/s[210°]$ (to quadrant bearing)



b) $d = 25m[N30°W]$ (to true bearing)