Day2: 2.1 – *The Remainder Theorem* (SYNTHETIC DIVISION)

SYNTHETIC DIVISION (can be used instead of long division unless the question asks you to use long division)

When dividing by a ***binomial*** of the form x – p, division is simplified by using a process called synthetic division. It is derived from the long division procedure by working only with numerical coefficients.

a) $(3x^3+2x^2-11-12x) \div (x+1)$

$$\begin{array}{r|rrrr} -1 & 3 & 2 & -12 & -11 \\ & & -3 & 1 & 11 \\ \hline & 3 & -1 & -11 & 0 \end{array}$$

$\underbrace{3 \quad -1 \quad -11}$ → Quotient -

$\therefore \dfrac{3x^3+2x^2-11-12x}{x+1} = 3x^2-x-11.$

OR

$3x^3+2x^2-11-12x = (3x^2-x-11)(x+1)$

b) $(x^3 - 2x^2 - 33x + 9\cancel{0}1) \div (x-5)$

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -33 & +9\cancel{0}1 \\ & & 5 & 15 & -90 \\ \hline & 1 & 3 & -18 & 1 \end{array}$$

$\therefore (x^3-2x^2-33x+91)$

$= (x^2+3x^2-18)(x-5)+1$

c) $(6x^3+5x^2-11x-12) \div (2x+1)$ → $2x+1=0 \Rightarrow x=-\frac{1}{2}$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 5 & -11 & -12 \\ & & -3 & -1 & 6 \\ \hline & 6 & 2 & -12 & -6 \end{array}$$

$\therefore \; P(x) = 6x^3+5x^2-11x-12 = (6x^2+2x-12)(x+\frac{1}{2}) - 6$

divide by

$= (3x^2+x-6)(2x+1)-6$

please NOTE: If (2x+1) is a factor, then the quotient must be divided by 2

Similarly, if (ax-b) is a factor, when using synthetic division, quotient must be divided by 'a'

SYNTHETIC DIVISION WORKSHEET

- Don't forget ZERO Coefficients for missing degrees
- Solve the binomial divisor equal to zero.
- If zero value is a fraction, then divide all coefficients by denominator.

1) Perform the following divisions using Synthetic Division.
Is the binomial divisor a factor of the polynomial?

A. $(x^4+5x^3-11x^2-25x+29)\div(x+6)$

-6	1	5	-11	-25	29
	↓	-6	6	30	-30
	1	-1	-5	5	-1 Rem.

$P(x) = (x^3 - x^2 - 5x + 5)(x+6) - 1$

$(x+6)$ is NOT a factor since $R \neq 0$

B. $(3x^3-4x^2-17x+6)\div(3x-1)$

$\frac{1}{3}$	3	-4	-17	6
	↓	1	-1	-6
	3	-3	-18	0 ⇒ divide by 3 / divide Q by 3

$\therefore P(x) = (x^2 - x - 6)(3x-1)$

OR $P(x) = (3x^2 - 3x - 18)(x - \frac{1}{3})$

C. $(8v^5+32v^4+5v+20)\div(v+4)$

-4	8	32	5	20
	↓	-32	0	-20
	8	0	5	0

$\therefore P(v) = (8v^4 + 5v^2)(v+4)$

D. $(6x^3+5x^2-3x-2)\div(2x+1)$

$-\frac{1}{2}$	6	5	-3	-2
	↓	-3	-1	2
	6	2	-4	0 ⇒ divide the Q by 2

$\therefore P(x) = (3x^2 + 2x - 2)(2x+1)$

2) Completely FACTOR each polynomial given a known factor.
What are all of the zeros of the polynomial?

A. $x^3+9x^2+23x+15;\ x+5$

-5	1	9	23	15
	↓	-5	-20	-15
	1	4	3	0

Zeros: $\{-5, -3, -1\}$

$\therefore P(x) = (x^2+4x+3)(x+5)$
$= (x+3)(x+1)(x+5)$

B. $25x^3+150x^2+131x+30;\ 5x+3$

$-\frac{3}{5}$	25	150	131	30
	↓	-15	-81	-30
	25	135	50	0 ⇒ divide by 5

$\therefore P(x) = (5x^2+27x+10)(5x+3)$
$= (5x+2)(x+5)(5x+3) \Rightarrow x = \{-\frac{2}{5}, -5, -\frac{3}{5}\}$

C. $x^3-x^2-14x+24;\ x-3$

3	1	-1	-14	24
	↓	3	6	-24
	1	2	-8	0

$\therefore P(x) = (x^2+2x-8)(x-3)$
$= (x+4)(x-2)(x-3)$

$x = \{-4, 2, 3\}$

D. $6x^3+7x^2-1;\ 2x+1$

$-\frac{1}{2}$	6	7	0	-1
	↓	-3	-2	1
	6	4	-2	0

$\therefore P(x) = (3x^2+2x-1)(2x+1)$
$= (3x-1)(x+1)(2x+1)$

zeros: $\{-\frac{1}{2}, -1, \frac{1}{3}\}$