MCR3U-Day 5 ***Rational Expressions: ADDING and SUBTRACTING***

To add or subtract rational numbers we need to write them in an equivalent form with common denominators:
Example 1:

$$\frac{3}{5}+\frac{2}{7}$$

$$= \frac{3}{5}\times\frac{7}{7} + \frac{2}{7}\times\frac{5}{5} = \frac{21}{35} + \frac{10}{35}$$

$$= \frac{21+10}{35} = \frac{31}{35}$$

To add or subtract rational expressions we use the same steps:
Example 2:

$$\frac{2x-3}{4}+\frac{3x-1}{5}-\frac{x-5}{2} \qquad \textbf{LCD} = 20$$

$$= \frac{5(2x-3)+4(3x-1)-10(x-5)}{20}$$ → expand & simplify NUMERATOR only.

$$= \frac{10x-15+12x-4-10x+50}{20}$$

$$= \frac{12x+31}{20}$$

Example 3:

$$\frac{10x}{10x}\cdot\frac{5}{3x^2}-\frac{1}{2x}\cdot\frac{15x^2}{15x^2}+\frac{3}{5x^3}\cdot\frac{6}{6} \qquad LCD = 30x^3$$

$$= \frac{(10x)5-(15x^2)1+(6)3}{30x^3}$$

$$= \frac{50x-15x^2+18}{30x^3}, \quad x \neq 0$$

Example 4:

$$\frac{3x-12}{x^2-x-12}-\frac{2}{x^2+6x+9}-\frac{1}{x^2-4x-21}$$

LCD = ? We need to factor!

$(x-4)(x-7)(x+3)(x+3)$

$$=\frac{3x-12}{(x-4)(x+3)}\cdot\frac{(x+3)(x-7)}{(x+3)(x-7)}-\frac{2}{(x+3)(x+3)}\cdot\frac{(x-4)(x-7)}{(x-4)(x-7)}-\frac{1}{(x-7)(x+3)}\cdot\frac{x-4}{x-4}\cdot\frac{x+3}{x+3}$$

$$=\frac{(3x-12)[(x+3)(x-7)]-2[(x-4)(x-7)]-1[(x-4)(x+3)]}{(x-4)(x-7)(x+3)(x+3)}$$

$$=\frac{(3x-12)[x^2-4x-21]-2[x^2-11x+28]-1[x^2-x-12]}{(x-4)(x-7)(x+3)(x+3)}$$

$$=\frac{3x^3-12x^2-63x-12x^2+48x+252-2x^2+22x-56-x^2+x+12}{(x-4)(x-7)(x+3)(x+3)}$$

$$=\frac{3x^3-27x^2+8x+264}{(x-4)(x-7)(x+3)^2},\ x\neq 4,7,-3$$

Practice:

1) $\dfrac{5}{(x+1)(x+2)}-\dfrac{7}{(x+2)(x-4)}$

2) $\dfrac{6}{x+4}+\dfrac{5}{x}$

3) $\dfrac{7}{2y-1}-\dfrac{3}{1-2y}$

4) $7+\dfrac{3m}{m-4}-\dfrac{m}{m+2}$

5) $\dfrac{2x}{x-y}-\dfrac{3y}{x+y}+1$

6) $\dfrac{1+\frac{2}{x}-\frac{3}{x^2}}{2-\frac{1}{x}-\frac{1}{x^2}}$

Homework: p. 128 #1c, 6b,7e,8b,9d,10d, 11

Practice

① $\frac{5}{(x+1)(x+2)} - \frac{7}{(x+2)(x-4)}$

$= \frac{5(x-4)-7(x+1)}{(x+1)(x+2)(x-4)}$

$= \frac{5x-20-7x-7}{(x+1)(x+2)(x-4)} = \frac{-2x-27}{(x+1)(x+2)(x-4)}, \quad x \neq -1, -2, 4$

② $\frac{6}{(x+4)} + \frac{5}{x}$

$= \frac{6x+5(x+4)}{(x+4)(x)} = \frac{11x+20}{(x+4)(x)}, \quad x \neq -4, 0$

③ $\frac{7}{2y-1} - \frac{3}{1-2y}$ NOTE: $1-2y = -(2y-1)$

$= \frac{7}{2y-1} + \frac{3}{2y-1} = \frac{10}{2y-1}, \quad y \neq \frac{1}{2}$

④ $7 + \frac{3m}{m-4} - \frac{m}{m+2}$

$= \frac{7(m-4)(m+2)+3m(m+2)-m(m-4)}{(m-4)(m+2)}$

$= \frac{7[m^2-2m-8]+3m^2+6m-m^2+4m}{(m-4)(m+2)}$

$= \frac{9m^2-4m-56}{(m-4)(m+2)}, \quad m \neq 4, -2$

(5) $\dfrac{2x}{x-y} - \dfrac{3y}{x+y} + 1$

$$= \frac{2x(x+y) - 3y(x-y) + 1(x+y)(x-y)}{(x+y)(x-y)}$$

$$= \frac{2x^2 + 2xy - 3xy + 3y^2 + x^2 - y^2}{(x+y)(x-y)}$$

$$= \frac{3x^2 + 2y^2 - xy}{(x+y)(x-y)} = \frac{3x^2 - xy + 2y^2}{(x+y)(x-y)}, \quad x \neq \pm y$$

(6) $\dfrac{1 + \frac{2}{x} + \frac{3}{x^2}}{2 - \frac{1}{x} - \frac{1}{x^2}}$

$$= \left(\frac{x^2 + 2x - 3}{x^2}\right) \div \left(\frac{2x^2 - x - 1}{x^2}\right)$$

$$= \frac{x^2 + 2x - 3}{(x^2)} \times \frac{x^2}{(2x^2 - x - 1)}$$

$$= \frac{(x+3)(x-1)(x^2)}{(x^2)(2x+1)(x-1)}$$

$$= \frac{x+3}{2x+1}, \quad x \neq 0, -\frac{1}{2}, 1$$