Day 2 - Review Continued...

Quadratic Functions:

- Degree of 2. Graph is a parabola. First differences are not the same but second differences are the same / constant.
- Forms
  - Standard Form $y=ax^2+bx+c, a\neq 0$. Y-intercept is c.
  - Vertex Form $y=a(x-h)^2+k, a\neq 0$. Vertex is (h,k).
  - Factored Form $y=a(x-r)(x-s)$. Zeroes are $x=r$ and $x=s$.

Note: To convert Standard Form to Vertex Form, we need to complete the square.

Example: Complete the square for $y = -2x^2 - 12x + 3$ and state the vertex.

$y = (-2x^2 - 12x) + 3$ factor -2

$= -2(x^2 - 6x) + 3$ add/subtract $\left(\frac{\text{middle term}}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = 9$

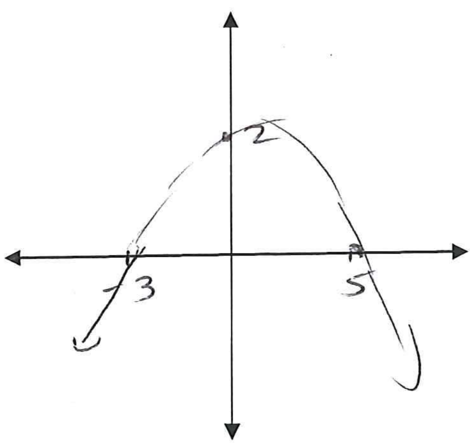
$= -2(x^2 - 6x + 9 - 9) + 3$ bring -9 outside after multiplying by -2

$= -2(x^2 - 6x + 9) + 3 + 18$

$= -2(x-3)^2 + 21$

The vertex is $(3, 21)$.

Example: Determine the equation of a quadratic function that has x-intercepts at -3 and 5 and has a y-intercept at 2.



$y = a(x+3)(x-5)$

Sub in $x=0$ $y=2$ to solve for a

$2 = a(0+3)(0-5)$

$a = \frac{2}{-15}$

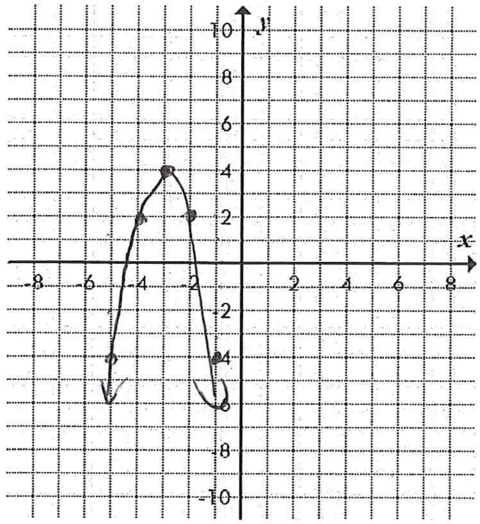
$\therefore y = -\frac{2}{15}(x+3)(x-5)$ → FACTORED FORM

Expand and simplify if standard form required.

Domain and Range:

- Domain: The set of values of independent variable for which a function or relation is defined.
- Range: The set of values of dependent variable of a function or relation.

Example: Graph the parabola and state the domain and range of $y = -2(x+3)^2 + 4$.



Vertex: $(-3, 4)$

Step pattern: $-2(1,3,5,7) = -2, -6, -10, -14$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \leq 4\}$.

Transformations:

$$y = af(k(x-d)) + c$$

vertical stretch or compression ($a < 0$: reflection in x-axis)

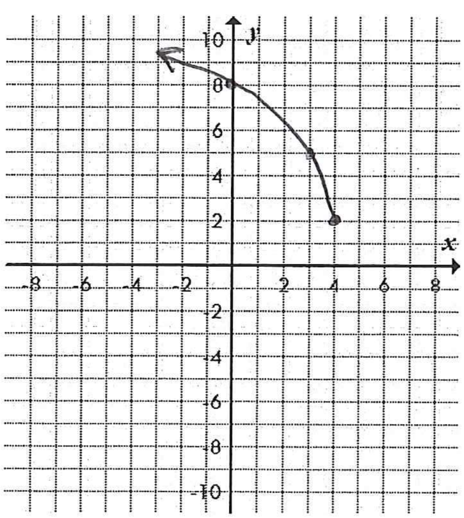
horizontal compression/stretch ($k < 0$: reflection in y-axis)

horizontal translation right/left

vertical translation up/down.

Mapping Notation: $(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$

Example: $y = 3\sqrt{-(x-4)} + 2$ $a = 3$ $k = -1$ $d = 4$ $c = 2$



$(x,y) \rightarrow \left(\frac{x}{k} + d, ay + c\right) = (-x + 4, 3y + 2)$

$(0,0) \rightarrow (4,2)$

$(1,1) \rightarrow (3,5)$

$(4,2) \rightarrow (0,8)$

$(9,3) \rightarrow (-5,11)$

Domain: $\{x \in \mathbb{R} \mid x \leq 4\}$

Range: $\{y \in \mathbb{R} \mid y \geq 2\}$

Day 2 - Prerequisite Skills Worksheet

1. Determine each value for the function $f(x) = -4x + 7$.

a. $f(0)$

$= -4(0)+7$

$= 7$

b) $f(-1)$

$= -4(-1)+7$

$= 11$

c) $f(-2x)$

$= -4(-2x)+7$

$= 8x+7$

2. Determine each value for the function $f(x) = 2x^2 - 3x + 1$.

a. $f(3)$

$= 2(3)^2 - 3(3)+1$

$= 18-9+1$

$= 10$

b) $f(-1)$

$= 2(-1)^2 - 3(-1)+1$

$= 2+3+1$

$= 6$

c) $3f(2x)$

$= 3[2(2x)^2 - 3(2x)+1]$

$= 3[8x^2 - 6x+1]$

$= 24x^2 - 18x+3$

3. State the slope and y-intercept of each line.

a. $y = 3x + 2$

$m=3$

$b=2$

b) $5x - y + 7 = 0$

$y=5x+7$

$m=5 \quad b=7$

c) $-(x + 4) = 2(y - 3)$

$-x-4 = 2y-6$

$2y = -x-4+6$

$2y = -x+2$

$y = -\frac{1}{2}x+1$

$m = \frac{-1}{2}$

$b=1$

4. Determine an equation for the line that satisfies each set of conditions.

a. $m = 3$ and $b = 5$

$y = 3x+5$

b) $m = -4$ and the line passes through $(7, 3)$

$y = -4(x-7)+3$

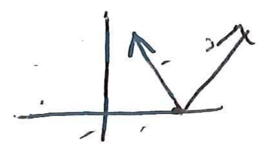
$= -4x+31$

5. Use finite differences to determine if the function is linear, quadratic, or neither.

x	y	
-2	-7	
-1	-5	2
0	-3	2
1	-1	2
2	1	2

function is LINEAR.
since first differences are constant

6. State the domain and range of each function. Justify your answer.



a. $y = 2(x - 3)^2 + 1$

min

$\{x \in \mathbb{R}\}$

$\{y \in \mathbb{R} \mid y \geq 1\}$

b) $y = \frac{1}{x+5}$

$\{x \in \mathbb{R} \mid x \neq -5\}$

$\{y \in \mathbb{R} \mid y \neq 0\}$

c) $y = |x-2|$

$\{x \in \mathbb{R}\}$

$\{y \in \mathbb{R} \mid y \geq 0\}$

7. Determine the equation of a quadratic function that satisfies each set of conditions.

a. x-intercepts 1 and -1, y-intercept 3

$y = a(x-1)(x+1)$ Sub $x=0$, $y=3$

$3 = a(-1)(1)$

$a = -3$

$\therefore y = -3(x-1)(x+1)$

b) x-intercept 3, and passing through point $(1, -2)$

$y = a(x-3)^2$

$-2 = a(1-3)^2$

$a = \frac{-2}{4} = \frac{-1}{2}$

$\therefore y = -\frac{1}{2}(x-3)^2$

$= -\frac{1}{2}(x^2 - 6x+9)$

$= -\frac{1}{2}x^2 + 3x - 3$

8. Determine the x-intercepts, the vertex, the direction of opening, and the domain and range of each quadratic function. Then, graph the function.

a. $y = (x+6)(2x-5)$

$x = \{-6, 5/2\}$ opens up

$h = \frac{-6+\frac{5}{2}}{2} = \frac{-7}{4}$

$k = (-\frac{7}{4}+6)(2(-\frac{7}{4})-5) = \frac{-289}{8}$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \geq -\frac{289}{8}\}$

b) $y = -2(x-4)^2 + 8 \rightarrow$ vertex $(4, 8)$

$D = \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R} \mid y \leq 8\}$

x-ints: $0 = -2(x-4)^2 + 8$

$4 = (x-4)^2$

$x - 4 = \pm 2 \Rightarrow x = 2, 6$

9. Identify each transformation of the function $y = f(x)$ as a vertical or horizontal translation, a stretch or compression, or a reflection in the x-axis or y-axis, or any combination of these.

a. $y = -4f(x)$	b. $y = \frac{1}{3}f(x)$	c. $y = f(2x)$	d. $y = f\left(-\frac{1}{3}x\right)$	e. $y = f(-x)$
• reflection in x-axis • Vertically stretched by a factor of 4	• vertically compressed by a factor of 1/3	• horizontally compressed by factor of ½	• reflection in y-axis • horizontally stretched by a factor of 3	• reflection in y-axis

Answers:

1.

a. 7 b) 11 c) $8x + 7$

2.

a. 10 b) 6 c) $24x^2 - 18x + 3$

3.

a. $m = 3, b = 2$ b) $m = 5,\ b = 7$ c) $m = -\frac{1}{2},\quad b = 1$

4.

a. $y = 3x + 5$ b) $y = -4x + 31$

5. Linear

6.

a. $\{x \in R\}, \{y \in R, y \geq 1\}$ b) $\{x \in R\}, x \neq 5\}, \{y \in R, y \neq 0\}$ c) $\{x \in R\}, \{y \in R, y \geq 0\}$

7.

a. $y = -3(x+1)(x-1)$ b) $y = -\frac{1}{2}x^2 + 3x - 3$

8.

a. x-intercepts $-6, \frac{5}{2}$; vertex $\left(-\frac{7}{4}, -\frac{289}{8}\right)$; opens up; $\{x \in R\}, \left\{y \in R, y \geq -\frac{289}{8}\right\}$
Check graph with the geogebra or desmos website.

b. x-intercepts: 2 , 6 ; vertex $(4, 8)$; opens down; $\{x \in R\}, \{y \in R, y \leq 8\}$
Check graph with the geogebra or desmos website.

9.

a. Vertical stretch by a factor of 4, reflection in x-axis
b. Vertical compression by a factor of $\frac{1}{3}$
c. Horizontal compression by a factor of $y = f(2x)$
d. Horizontal Stretch by a factor of 3, reflection in y-axis
e. Reflection in the y-axis