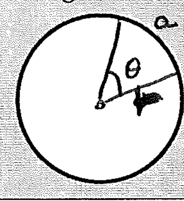
Day 2 - Radian Measure

In the past, we have worked exclusively with degrees as our unit of measurement for angles. An alternative measurement system uses **radians**.

The measure of an angle θ is defined by the length, a, of the arc that subtends the angle divided by the radius of the circle



$\theta = 1$
$a = r$

An **angle θ** (in radians) is equal to **the arc length (a)** divided by the **radius (r)** of the circle

$$\theta = \frac{a}{r}$$

For one complete revolution, the length of the arc equals the circumference of the circle, $2\pi r$

$$\theta = \frac{2\pi\ r}{r}$$

One complete revolution (360°) measures 2π radians, therefore $2\pi = 360°$ or $\pi = 180°$

To convert from:

Radians to degrees: multiply by $\frac{180}{\pi}$

Degrees to radians: multiply by $\frac{\pi}{180}$.

EX 1 - Convert the following:

10° to radians

$= (10)\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{18}$ rad

220° to radians

$= 220\left(\frac{\pi}{180}\right)$

$= \frac{22}{18}\pi$

$= \frac{11\pi}{9}$ rad.

$\frac{\pi}{6}$ to degrees

$= \frac{\pi}{6} \cdot \frac{180}{\pi}$

$= 30°$

$\frac{2\pi}{10}$ to degrees

$= \frac{2\pi}{10}\left(\frac{180}{\pi}\right)$

$= 36°$

OR 2π is 360

$360/10 = 36°$

EX 2 - Liah chooses a camel to ride on a carousel. The camel is located 9 m from the centre of the carousel. If the carousel turns through an angle of $\frac{5\pi}{6}$, determine the length of the arc traveled by the camel.

$a = ?$

$r = 9m$

$\theta = \frac{5\pi}{6}$

$a = r\theta$

$= 9\left(\frac{5\pi}{6}\right)$

$= \frac{45\pi}{9} = 5\pi$ metres.

As an object rotates, its angular displacement changes with respect to time.
Angular velocity $= \dfrac{\theta}{t}$

EX 3 - The angular velocity of a rotating object is the rate at which the central angle changes with respect to time. The hard disk in a personal computer rotates at 7200 rpm (revolutions per minute). Determine its angular velocity, in:

a) Degrees per second

$$= \frac{(7200)(360)}{60}$$

$$= 43200^\circ/\text{sec}$$

b) Radians per second

$$= \frac{(7200)(2\pi)}{60}$$

$$= 240\pi \text{ rad}/\text{sec}$$

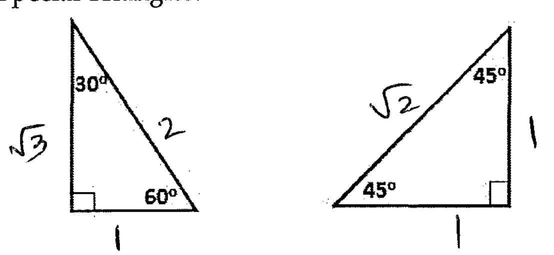
Trigonometry Essential Skills Review

Recall: S^{O}_{H} C^{A}_{H} T^{O}_{A}

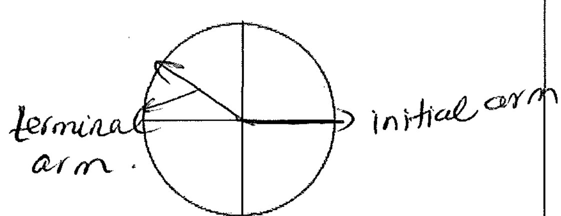
Primary trig ratios: $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\quad cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\quad tan\theta = \dfrac{\text{opp}}{\text{adj}}$

Secondary trig ratios: $\csc\theta = \dfrac{1}{\sin\theta}$ $\quad \sec\theta = \dfrac{1}{\cos\theta}$ $\quad \cot\theta = \dfrac{1}{\tan\theta}$

Recall:
Special Triangles:



Recall:
Angles in standard position:



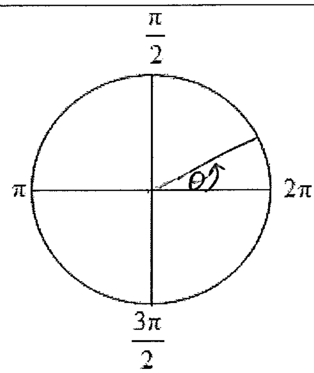
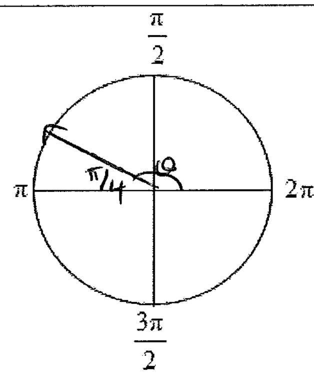
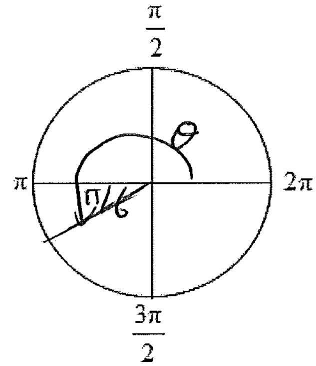
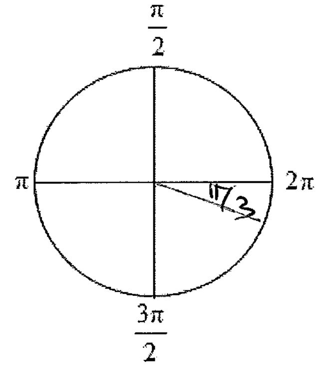
Skill practice - Rationalizing the denominator:

$\dfrac{3}{\sqrt{2}}$ has an *irrational* denominator (**root in the denominator**).

This is improper form. We <u>rationalize</u> the denominator by: $\dfrac{3}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{2}$

$$\frac{3}{1+\sqrt{2}} \cdot \frac{(1-\sqrt{2})}{(1-\sqrt{2})} = \frac{3-3\sqrt{2}}{1-2} = \frac{3-3\sqrt{2}}{-1} = -3+3\sqrt{2}$$

Recall: We can find **principal angles** given the specified quadrant and **related acute angle.** We can also measure these rotations in radians with special angles $\frac{\pi}{6}$ (30°), $\frac{\pi}{4}$ (45°), $\frac{\pi}{3}$ (60°).

Quadrant I - $\frac{\pi}{3}$	Quadrant II - $\frac{\pi}{4}$	Quadrant III - $\frac{\pi}{6}$	Quadrant IV - $\frac{\pi}{3}$
			
$\theta = \frac{\pi}{3}$	$\theta = \pi - \frac{\pi}{4}$ $= \frac{3\pi}{4}$	$\theta = \pi + \frac{\pi}{6}$ $= \frac{7\pi}{6}$	$\theta = 2\pi - \pi/3$ $= \frac{5\pi}{3}$

Practice Questions:

1. Convert the following to radians.

a. 40° $= 40\left(\frac{\pi}{180}\right)$ $= 2\pi/9$ rad

b. 60° $\cdot \frac{\pi}{180}$ $= \frac{\pi}{3}$

c. 140° $= 140\left(\frac{\pi}{180}\right)$ $= \frac{7\pi}{9}$

d. 210° $= \frac{210\pi}{180}$ $= \frac{7\pi}{6}$

2. Convert the following to degrees

a. $\frac{5\pi}{6}$ $= \frac{5(180)}{6} = 150°$

b. $\frac{3\pi}{4} = \frac{3(180)}{4}$ $= 135°$

c. $\frac{7\pi}{3}$ $= \frac{7(180)}{3}$ $= 420°$

d. $\frac{11\pi}{4}$ $= \frac{11(180)}{4}$ $= 495°$

3. Find each principal angle for the specified quadrant and RAA (related acute angle)

a. Q2: $\frac{\pi}{6}$

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ rad

b. Q3: $\frac{\pi}{4}$

$\theta = \pi + \frac{\pi}{4}$ $= \frac{5\pi}{4}$ rad

c. Q4: $\frac{\pi}{3}$

$\theta = 2\pi - \frac{\pi}{3}$ $= \frac{5\pi}{3}$

4. Rationalize each denominator:

a. $\frac{4}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ $= \frac{4\sqrt{7}}{7}$

b. $\frac{12}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$ $= \frac{12\sqrt{6}}{6} = 2\sqrt{6}$

c. $\frac{8\sqrt{5}}{\sqrt{3}} = \frac{8\sqrt{5}\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $= \frac{8\sqrt{15}}{3}$