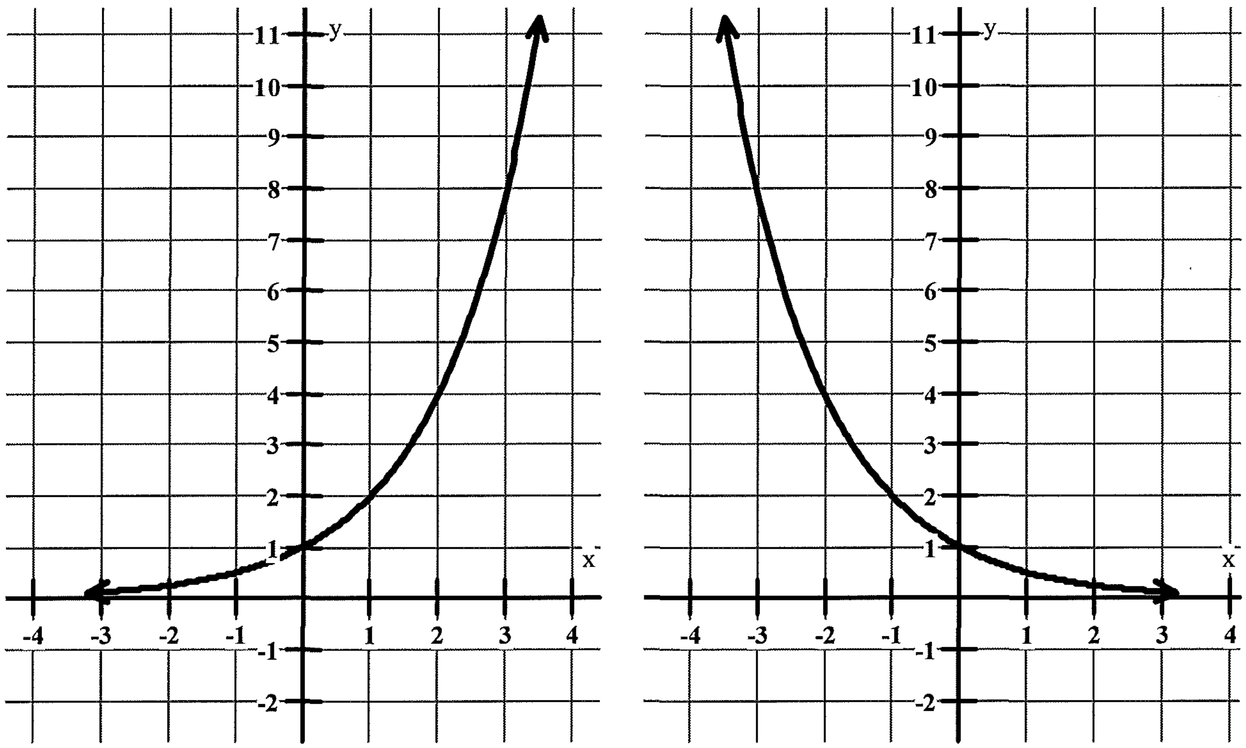
Day 2: 6.1 – Exponential Functions

Recall: **Exponential functions** are equations written in the form: $y = b^x$, where $b > 0$ and $b \neq 1$

- Functions such as $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ are examples of **exponential functions.**
- These types of function can model many different phenomena, including population growth and the cooling of a liquid.

Review: Properties of Exponential Functions

Exponential **Growth:** | Exponential **Decay**



- This function is increasing on its' domain where $b > 1$
- The greater the value of b, the faster the growth

- This function is decreasing on its' domain where $0 < b < 1$
- The lower the value of b, the faster the decay

- The x-axis, $y = 0$, is the horizontal asymptote.
- The y-intercept is 1.
- The domain is $\{x \in \mathbb{R}\}$
- The range is $\{y \in \mathbb{R} \mid y > 0\}$

Review - Working with Exponent Laws

1. $\dfrac{-8x^{-4}}{(2x^{-2})^2}$

$= \dfrac{-8x^{-4}}{4x^{-4}}$

$= -2$

2. $\left(\dfrac{25m^{-1}n^5}{75m^2n^2}\right)^{-3}$

$= \left(\dfrac{75m^2n^2}{25m^{-1}n^5}\right)^3$

$= \left(3m^3n^{-3}\right)^3$

$= \dfrac{27m^9}{n^9}$

3. $\left(27a^{-3}b^{12}\right)^{\frac{1}{3}}$

$= \left(3^3a^{-3}b^{12}\right)^{\frac{1}{3}}$

$= 3a^{-1}b^4$

$= \dfrac{3b^4}{a}$

4. $\left(\dfrac{\sqrt[5]{x^8}}{\sqrt{x^3}}\right)^3$

$= \left(\dfrac{x^{\frac{8}{5}}}{x^{\frac{3}{2}}}\right)^3$

$= \left(x^{\frac{8}{5}-\frac{3}{2}}\right)^3$

$= \left(x^{\frac{16-15}{10}}\right)^3$

$= \left(x^{\frac{1}{10}}\right)^3$

$= x^{\frac{3}{10}}$

Exponential functions and expressions can be expressed in different ways by changing the base.

Examples:

Write 9 with a base of 3	Write 27^{2x-1} with a base of 3	Write $\left(\frac{1}{8}\right)^{x}$ with a base of 2	Write 1 with a base of 35
$9 = 3^{2}$	$= \left(3^{3}\right)^{2x-1}$ $= 3^{6x-3}$	$\left(\frac{1}{8}\right)^{x} = \left(2^{-3}\right)^{x}$ $= 2^{-3x}$	$1 = \left(35\right)^{0}$

Changing the base of *one or more* exponential expressions is a useful technique for **<u>solving exponential equations</u>**

To solve exponential equations:

1. **Write each side of the equation with a common base**
2. **Set the exponents equal to each other**
3. **Solve for the unknown variable**

EX 1 – Solve each equation by determining a common base:

a) $4^{2x} = 16^{2x-1}$

$4^{2x} = \left(4^{2}\right)^{2x-1}$

$2x = 4x - 2$

$-2x = -2$

$\boxed{x = 1}$

c) $5^{3+x} = \frac{1}{25}$

$5^{x+3} = 5^{-2}$

$x + 3 = -2$

$\boxed{x = -5}$

b) $9^{3x+1} = 27^{x}$

$\left(3^{2}\right)^{3x+1} = \left(3^{3}\right)^{x}$

$6x + 2 = 3x$

$3x = -2$

$\boxed{x = \frac{-2}{3}}$

d) $\frac{243^{2x-1}}{9^{x-3}} = 81^{x}$

$\frac{\left(3^{5}\right)^{2x-1}}{\left(3^{2}\right)^{x-3}} = \left(3^{4}\right)^{x}$

$3^{10x-5} = 3^{4x}\, 3^{2x-6}$

$3^{10x-5} = 3^{6x-6}$

$10x - 5 = 6x - 6$

$4x = -1 \Rightarrow x = -1/4$

Practice - Solve for x algebraically by determining a common base:

a) $2^{5x+2} = 8^x \Rightarrow 2^{5x+2} = (2^3)^x$

$5x+2 = 3x$

$2x = -2$

$x = -1$

b) $8^x = 1$

$8^x = 8^0$

$\boxed{x = 0}$

c) $\left(\frac{2}{3}\right)^{x+5} = \frac{16}{81} = \left(\frac{2}{3}\right)^4$

$x+5 = 4$

$\boxed{x = -1}$

d) $125^{x-2} = 25^{2x+1} \Rightarrow (5^3)^{x-2} = (5^2)^{2x+1}$

$3x-6 = 4x+2$

$-x = 8$

$x = -8$

e) $5^{x^2-3x} = 5^{2x-4}$

$x^2 - 3x = 2x - 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = \{1, 4\}$

f) $3(5^{x+1}) = 15$

$5^{x+1} = 5$

$x+1 = 1$

$x = 0$

Homework:

1. Solve for x algebraically by determining a common base:

a. $12^{3x-9} = 1$

$12^{3x-9} = 12^0$

$3x - 9 = 0$

$x = 3$

b. $\left(\frac{1}{9}\right)^{x+2} = 27^{x+1}$

$(3^{-2})^{x+2} = (3^3)^{x+1}$

$-2x - 4 = 3x + 3$

$-5x = 7$

$\boxed{x = -\frac{7}{5}}$

c. $4^{2x} = 16^{2x-1}$

$4^{2x} = (4^2)^{2x-1}$

$2x = 4x - 2$

$-2x = -2$

$\boxed{x = 1}$

d. $2^{2x} = 8^{x+3}$

$2^{2x} = (2^3)^{x+3}$

$2x = 3x+9$

$-x = 9$

$\boxed{x=-9}$

e. $27^{3m-1} = 9^{m+2}$

$(3^3)^{3m-1} = (3^2)^{m+2}$

$9m-3 = 2m+4$

$7m = 7$

$\boxed{m=1}$

f. $5^{2n+1} = \frac{1}{125}$

$5^{2n+1} = 5^{-3}$

$2n+1 = -3$

$2n = -4$

$\boxed{n=-2}$

g. $625^{4y+3} = 25^{y-2}$

$(5^4)^{4y+3} = (5^2)^{y-2}$

$16y+12 = 2y-4$

$14y = -16$

$y = \frac{-16}{14} = \frac{-8}{7}$

h. $\left(\frac{2^{2x+1}}{2^{x-3}}\right) = 4$

$2^{2x+1-(x-3)} = 2^2$

$2^{x+4} = 2^2$

$x+4 = 2$

$\boxed{x=-2}$

i. $2(3^{x-2}) = 18$

$3^{x-2} = 9$

$3^{x-2} = 3^2$

$x-2 = 2$

$\boxed{x=4}$

j. $27(3^{3x+1}) = 3$

$3^{3x+1} = \frac{1}{9}$

$3^{3x+1} = 3^{-2}$

$3x+1 = -2$

$3x = -3$

$\boxed{x=-1}$

k. $(3^{2n})(81) = 27$

$(3^{2n})(3)^4 = 3^3$

$2n+4 = 3$

$2n = -1$

$\boxed{n=-\frac{1}{2}}$

l. $3^{-n} = 3^{3n}$

$-n = 3n$

$4n = 0$

$\boxed{n=0}$

2. Simplify using exponent laws:

a) $\left(\sqrt[3]{\sqrt[4]{x^6}}\right)^7$

$= \left[\left(\left(x^6\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right]^7$

$= \left[x^{6/12}\right]^7 = \left[x^{1/2}\right]^7$

$= x^{7/2}$

b) $\frac{(3x^2y^5)(10x^3y^{-4})}{6xy^3}$

$= \frac{30x^5y}{6xy^3}$

$= \frac{5x^4}{y^2}$

c) $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$

$= \left(\frac{4}{25}\right)^{\frac{3}{2}}$

$= \left(\frac{2}{5}\right)^3 = \frac{8}{125}$

Answers:
1. a) 3 b) $-\frac{7}{5}$ c) 1 d) -9 e) 1 f) -2 g) $-\frac{8}{7}$ h) -2 i) 4 j) -1 k) $-\frac{1}{2}$ l) -1
2. a) $x^{\frac{7}{2}}$ b) $\frac{5x^4}{y^2}$ c) $\frac{8}{125}$