

PRESENT VALUE

RECAP

Simple Interest Formula	Compound Interest Formula
$A = P(1 + rt)$	$A = P(1 + i)^n$
$P = \frac{A}{1 + rt}$	$P = \frac{A}{(1 + i)^n} = A(1 + i)^{-n}$

Another word for... $A =$ Amount is... Final / accumulated / Future value
 $P =$ Principal is ... deposit / present value.

PRESENT VALUE FORMULA

The compound interest formula $A = P(1 + i)^n$ can be rearranged to solve for P so that

$$P = \frac{A}{(1 + i)^n}$$

or written with a negative exponent

$$P = A(1 + i)^{-n}$$

EXAMPLE 1: INVESTMENTS

Ravi wants to invest enough money today to have \$5 500 for college tuition in two years. If he invests his money at 6% per year, compounded monthly, how much does he need to invest?

$$P = ?$$

$$A = 5500$$

$$i = \frac{0.06}{12} = 0.005$$

$$n = 2 \times 12 = 24$$

$$P = A(1 + i)^{-n}$$

$$= 5500(1.005)^{-24}$$

$$= \$41879.52$$

EXAMPLE 2: LOANS

Suppose you want to borrow \$200. A creditor will add interest to the principal and then give you a loan for the full amount (interest included). You then make payments until the entire loan is paid off.

Jamie took out a \$3 000 loan, due in four years. If interest is 5.7% per year, compounded semi-annually, how much should Jamie's creditor be willing to accept to pay off the loan today?

$$P = ?$$

$$A = 3000$$

$$i = \frac{0.057}{2}$$

$$n = 4 \times 2$$

$$= 8$$

$$P = 3000 \left(1 + \frac{0.057}{2}\right)^{-8}$$

$$= \$2396.00$$

PRESENT VALUE PRACTICE

Use the present value formula $P = A(1+i)^{-n}$ to solve the following problems.

1. A loan of \$5000, at 12% per year compounded monthly is due to be repaid in 3 years. How much is the present value (principal) of the loan?

$$P = ?$$

$$A = 5000$$

$$i = \frac{0.12}{12} = 0.01$$

$$n = 36$$

$$P = 5000 (1.01)^{-36}$$

$$= \$3494.62$$

2. How much money must Kerry invest today to have \$4000 in two years, at 12% per year, compounded quarterly?

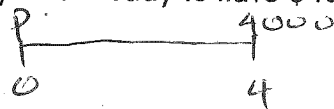
$$P = ?$$

$$A = 4000$$

$$i = 0.03$$

$$n = 2 \times 4$$

$$= 8$$



$$P = 4000 (1.03)^{-8}$$

$$= \$3157.64$$

3. Jenay will invest some money on July 3, her sixteenth birthday, at 4.5 per year, compounded monthly. How much should she invest if she wants to have \$10 000 on the November 3 following her eighteenth birthday?

$$P = ?$$

$$A = 10000$$

$$i = \frac{0.045}{12} =$$

$$n = 28$$

$$P = 10000 \left(1 + \frac{0.045}{12}\right)^{-28}$$

$$= 9005.01$$

4. An investment fund pays 6.3% per year, compounded monthly. How much should a 25-year-old woman invest in the fund to have \$50 000 by age 35?

$$P =$$

$$A = 50000$$

$$i = 0.00525$$

$$n = 120$$

$$P = 50000 (1 + 0.00525)^{-120}$$

$$= 26673.51$$

Compare Compounding Periods

5. Suppose you need \$5 000 in 4 years. How much money do you need to invest at 6.5% per year if the investment is compounded:

a) Yearly

$P =$

$$A = 5000$$

$$i = 0.065$$

$$n = 4$$

$$P = 5000(1 + 0.065)^{-4}$$

$$= 3886.62$$

b) Semi-annually

$P =$

$$A = 5000$$

$$i = 0.0325$$

$$n = 8$$

$$P = 5000(1 + 0.0325)^{-8}$$

$$= 3871.23$$

c) Monthly

$P =$

$$A = 5000$$

$$i = \frac{0.065}{12}$$

$$n = 48$$

$$P = 5000\left(1 + \frac{0.065}{12}\right)^{-48}$$

$$= 3857.96$$

d) Weekly

$P =$

$$A = 5000$$

$$i = \frac{0.065}{52}$$

$$n = 208$$

$$P = 5000\left(1 + \frac{0.065}{52}\right)^{-208}$$

$$= 3855.88$$

e) Daily

$P =$

$$A = 5000$$

$$i = \frac{0.065}{365}$$

$$n = 1460$$

$$P = 5000\left(1 + \frac{0.065}{365}\right)^{-1460}$$

$$= 3855.35$$