Prove the identity.

a) $\dfrac{1}{\sin^2 x}+\dfrac{1}{\cos^2 x}=\dfrac{1}{\sin^2 x\cos^2 x}$

b) $\tan x+\dfrac{1}{\tan x}=\dfrac{1}{\sin x\cos x}$

c) $\dfrac{1}{1-\cos x}+\dfrac{1}{1+\cos x}=\dfrac{2}{\sin^2 x}$

d) $(\sin x+\cos x)^2=1+2\sin x\cos x$

e) $(1-\cos^2 x)\left(1+\dfrac{1}{\tan^2 x}\right)=1$

f) $\dfrac{1+2\sin x\cos x}{\sin x+\cos x}=\sin x+\cos x$

g) $\dfrac{\sin x}{1-\cos x}-\dfrac{1+\cos x}{\sin x}=0$

h) $\sin^2 x-\sin^4 x=\cos^2 x-\cos^4 x$

i) $(1+\tan^2 x)(1-\cos^2 x)=\tan^2 x$

j) $\dfrac{\sin x-1}{\sin x+1}=\dfrac{-\cos^2 x}{(\sin x+1)^2}$

k) $\sin^4 x-\cos^4 x=\sin^2 x-\cos^2 x$

l) $\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x=1$

m) $\dfrac{4}{\cos^2 x}-5=4\tan^2 x-1$

n) $\dfrac{\cos x-\sin x-\cos^3 x}{\cos x}=\sin^2 x-\tan x$

o) $\dfrac{\sin^2 x-6\sin x+9}{\sin^2 x-9}=\dfrac{\sin x-3}{\sin x+3}$

p) $\dfrac{\tan x\sin x}{\tan x+\sin x}=\dfrac{\tan x-\sin x}{\tan x\sin x}$

a) $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$

$LS = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$ $\quad LCD = \sin^2 x \cos^2 x$

$= \frac{1 \cdot \cos^2 x}{\sin^2 x \cos^2 x} + \frac{1 \cdot \sin^2 x}{\cos^2 x \sin^2 x}$

$= \frac{\cos^2 x}{\sin^2 x \cos^2 x} + \frac{\sin^2 x}{\sin^2 x \cos^2 x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$

$= \frac{1}{\sin^2 x \cos^2 x}$

$LS = RS$ $\boxed{\text{QED}}$

b) $\tan x + \frac{1}{\tan x} = \frac{1}{\sin x \cos x}$

$LS = \tan x + \frac{1}{\tan x}$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ $\quad LCD = \sin x \cos x$

$= \frac{\sin x \sin x}{\cos x \sin x} + \frac{\cos x \cos x}{\sin x \cos x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$

$= \frac{1}{\sin x \cos x}$ $\quad LS = RS$ QED

c) $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}$

$LS = \frac{1}{1-\cos x} + \frac{1}{1+\cos x}$ $\quad LCD = (1-\cos x)(1+\cos x)$

$= \frac{1\,(1+\cos x)}{(1-\cos x)(1+\cos x)} + \frac{1\,(1-\cos x)}{(1+\cos x)(1-\cos x)}$

$= \frac{1+\cos x + 1 - \cos x}{1-\cos^2 x}$

$= \frac{2}{\sin^2 x}$

$LS = RS$ QED

d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

$LS = (\sin x + \cos x)^2$

$= \sin^2 x + 2\sin x \cos x + \cos^2 x$

$= 1 + 2\sin x \cos x$

$LS = RS$

QED

e) $(1-\cos^2 x)\left(1+\frac{1}{\tan^2 x}\right) = 1$

$= (1-\cos^2 x)\left(1+\frac{\cos^2 x}{\sin^2 x}\right)$

$= (\cancel{\sin^2 x})\frac{(\underbrace{\sin^2 x+\cos^2 x}_{1})}{\cancel{\sin^2 x}}$

$= 1$ LS = RS QED

f) $\frac{1+2\sin x\cos x}{\sin x+\cos x} = \sin x+\cos x$

LS = $\frac{1+2\sin x\cos x}{\sin x+\cos x}$ $(\sin x+\cos x)^2 = \underbrace{\sin^2 x+\cos^2 x}_{1}+2\sin x\cos x$

$= \frac{(\sin x+\cos x)^2}{(\sin x+\cos x)}$

$= (\sin x+\cos x)$ LS = RS QED

g) $\frac{\sin x}{1-\cos x} - \frac{1+\cos x}{\sin x} = 0$

LS ⇒ LCD $\sin x(1-\cos x)$

$\frac{\sin x\,(\sin x)}{(1-\cos x)(\sin x)} - \frac{(1+\cos x)(1-\cos x)}{\sin x\,(1-\cos x)}$

$= \frac{\sin^2 x - (1-\cos^2 x)}{(1-\cos x)(\sin x)}$

$= \frac{\sin^2 x - 1 + \cos^2 x}{(1-\cos x)(\sin x)}$ ∴ LS = RS

$= \frac{1-1}{\dots} = 0$ QED

h) $\underbrace{\sin^2 x - \sin^4 x}_{LS} = \underbrace{\cos^2 x - \cos^4 x}_{RS}$

$= \sin^2 x - (\sin^2 x)(\sin^2 x)$	$\cos^2 x - (\cos^2 x)(\cos^2 x)$
$= \sin^2 x(1-\sin^2 x)$	$= \cos^2 x(1-\cos^2 x)$
$= \sin^2 x(\cos^2 x)$	$= \cos^2 x(\sin^2 x)$

LS = RS

QED

i) $(1+\tan^2 x)(1-\cos^2 x) = \tan^2 x$

$= \left(1+\frac{\sin^2 x}{\cos^2 x}\right)(\sin^2 x)$

$= \left(\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}\right)\sin^2 x$

$= \left(\frac{\sin^2 x+\cos^2 x}{\cos^2 x}\right)\sin^2 x$

$= \frac{(1)\sin^2 x}{\cos^2 x}$ LS = RS QED

$= \tan x$

j) $\frac{\sin x-1}{\sin x+1} = \frac{-\cos^2 x}{(\sin x+1)^2}$

RS = $\frac{-(1-\sin^2 x)}{(\sin x+1)^2} = \frac{\sin^2 x-1}{(\sin x+1)^2} = \frac{(\sin x-1)\cancel{(\sin x+1)}}{(\sin x+1)^{\cancel{2}}}$

$= \frac{\sin x-1}{\sin x+1}$ ∴ LS = RS

QED

k) $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$

$LS = (\sin^2 x - \cos^2 x)\underbrace{(\sin^2 x + \cos^2 x)}_{1}$

$LS = \sin^2 x - \cos^2 x$

$LS = RS$ QED

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l) $\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = 1$

$LS = (\sin^2 x + \cos^2 x)^2$

$= (1)^2$

$= 1$

$\therefore LS = RS$ QED

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m) $\frac{4}{\cos^2 x} - 5 = 4\tan^2 x - 1$

$RS = 4\frac{\sin^2 x}{\cos^2 x} - 1 \quad LCD = \cos^2 x$

$= \frac{4\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$

$= \frac{4\sin^2 x - \cos^2 x}{\cos^2 x}$

$= \frac{4(1-\cos^2 x) - \cos^2 x}{\cos^2 x}$

$= \frac{4 - 4\cos^2 x - \cos^2 x}{\cos^2 x}$

$= \frac{4 - 5\cos^2 x}{\cos^2 x}$

RS

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$LS = \frac{4}{\cos^2 x} - 5 \quad LCD = \cos^2 x$

$\frac{4}{\cos^2 x} - \frac{5(\cos^2 x)}{(\cos^2 x)}$

$= \frac{4 - 5\cos^2 x}{\cos^2 x}$

$LS = RS$ QED.

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n) $\frac{\cos x - \sin x - \cos^3 x}{\cos x} = \sin^2 x - \tan x$

$LS = \frac{\cos x - \cos^3 x - \sin x}{\cos x}$

$= \frac{\cos x(1-\cos^2 x) - \sin x}{\cos x}$

$= \frac{\cos x(\sin^2 x) - \sin x}{\cos x}$

$RS = \sin^2 x - \tan x$

$= \sin^2 x - \frac{\sin x}{\cos x} \quad LCD = \cos x$

$RS = \frac{\sin^2 x\cos x - \sin x}{\cos x}$

$LS = RS$ QED

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o) $\frac{\sin^2 x - 6\sin x + 9}{\sin^2 x - 9} = \frac{\sin x - 3}{\sin x + 3}$

LS

$\frac{(\sin x - 3)^{\cancel{2}}}{(\cancel{\sin x - 3})(\sin x + 3)} = \frac{\sin x - 3}{\sin x + 3}$

$LS = RS$ QED

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q) $\frac{\tan x\sin x}{\tan x + \sin x} = \frac{\tan x - \sin x}{\tan x\sin x}$

$LS = \frac{\frac{\sin x}{\cos x}\cdot\sin x}{\frac{\sin x}{\cos x} + \sin x} = \frac{\frac{\sin^2 x}{\cos x}}{\frac{\sin x + \sin x\cos x}{\cos x}} = \frac{\sin^{\cancel{2}} x}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{\cancel{\sin x}(1+\cos x)}$

$= \frac{\sin x}{1+\cos x} = \frac{\sin x}{\frac{\sin^2 x}{1-\cos x}} = \cancel{\sin x}\cdot\frac{1-\cos x}{\sin^{\cancel{2}} x}$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = (1-\cos x)(1+\cos x)$

$\frac{\sin^2 x}{1-\cos x} = 1 + \cos x$

$= \frac{1-\cos x}{\sin x}$

common

$RS = \frac{\frac{\sin x}{\cos x} - \sin x}{\frac{\sin x}{\cos x}\cdot\sin x} = \frac{\frac{\sin x - \sin x\cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}} = \frac{\sin x - \sin x\cos x}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{\sin^2 x}$

factor

$= \frac{\cancel{\sin x}(1-\cos x)}{\sin^{\cancel{2}} x} = \frac{1-\cos x}{\sin x}$

$LS = RS$ QED