Day1: 7.1 Vectors as Forces

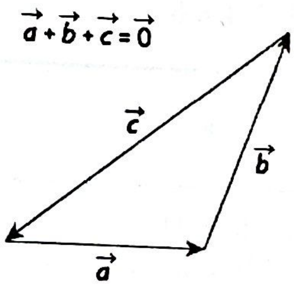
A **force** causes an object to undergo acceleration. For example, forces push you back in your seat when the car you are in accelerates.

The magnitude of force is measured in Newtons (N). At the earth's surface, gravity causes objects to accelerate at a rate of approximately **9.8 m/s²**.

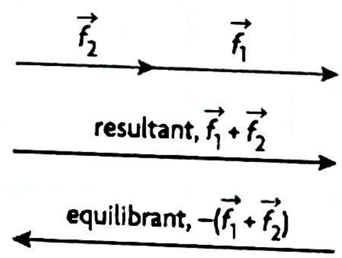
Forces are vectors. The single force $\vec{F}$, the **resultant force**, that has the same effect as all the forces acting together can be found by vector addition. $\vec{F} = \vec{F}_1 + \vec{F}_2$

When an object is in a state of **equilibrium** (a state of rest or a state of uniform motion), the net force is zero or $\vec{R} = 0$. For example, steady speed.

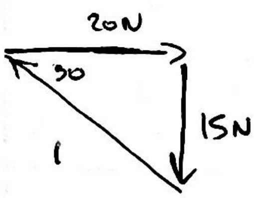
The equilibrant, $\vec{E}$, is the opposite force of the resultant force $\vec{F}$, it is the force that would counterbalance the resultant force. $\vec{E} = -\vec{F}$ and $|\vec{E}| = |\vec{F}|$



Three non-collinear forces



Find an equilibrant for the following system of forces.

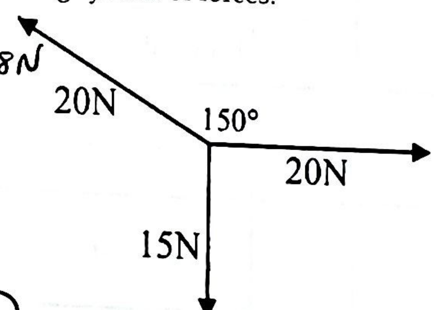


$R_x = 20 - 20\cos 30° = 2.68N$

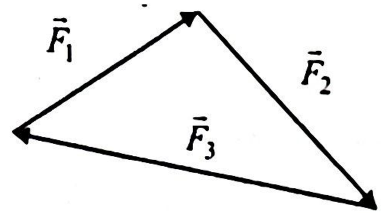
$R_y = 20\sin 30 - 15 = 5N$

$\vec{R} = (2.68, -5)$ N $\therefore \vec{E} = -\vec{R}$

$= (-2.68, 5)$



Given three forces on a plane, a state of equilibrium is maintained if a triangle can be formed with the three forces. This can only be done if the triangle inequality holds true that the sum of any two sides must be greater than or equal to the third side.



Ex 1: Which of the following sets of forces acting on an object could produce equilibrium?

a) 13N, 27N, 14N

$13 + 14 \geq 27$

$\therefore$ The set of forces can produce equilibrium

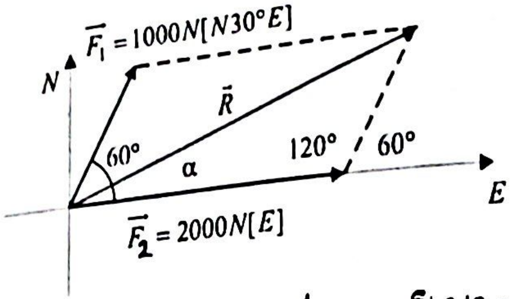
b) 12N, 26N, 13N

$12 + 13 < 26$

NO. They can not be in equilibrium

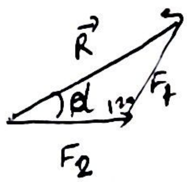
Ex 2: Find the magnitude and direction of the resultant and equilibrant of a system of forces of 2000N and 1000N acting at an angle of 60° to each other. USE COSINE LAW $a^2 = b^2 + c^2 - 2bc\cos A$

$$|R|^2 = 1000^2 + 2000^2 - 2(1000)(2000)\cos 120$$



$$|R|^2 = 7000000$$

$$|R| \doteq 2645.75\ N$$

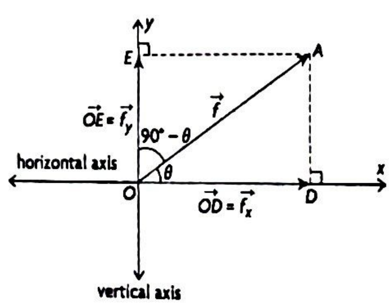


$$\frac{\sin\alpha}{1000} = \frac{\sin 120}{2645.75}$$

$$\angle\alpha = 19°$$

Algebraic Resultant force:

To <u>resolve</u> a vector means taking a single force and decomposing it into two components. A vector can be <u>resolved</u> into its corresponding horizontal and vertical components by creating a right triangle with the given vector. The magnitudes of the vertical and horizontal components can be found using primary trigonometric ratios and a given angle.



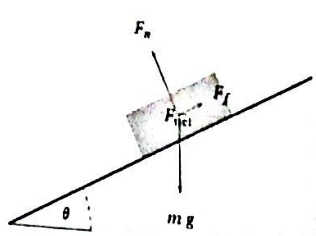
$$R_x = F_{1x} + F_{2x} + \ldots + F_{nx}$$
$$R_y = F_{1y} + F_{2y} + \ldots + F_{ny}$$

$$\|\vec{R}\| = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\tan\theta = \frac{R_y}{R_x}$$

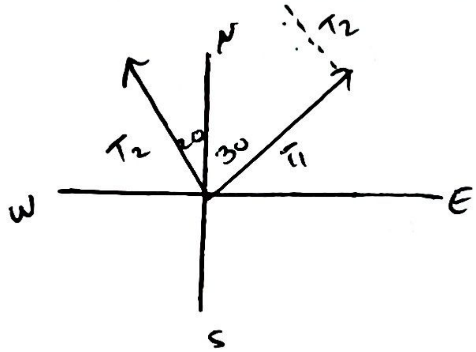
Ex 3: In order to keep a 250 kg crate from sliding down a ramp inclined at 25°, the force that acts parallel to the ramp must have a magnitude of at least how many Newtons? ↙ x

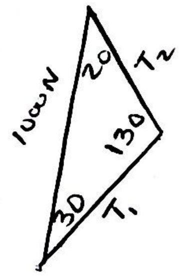
$$F = |mg| = (250)(9.8) = 2450\ N$$



$$|x| = F\sin 25°$$
$$= (2450 \times \sin 25°)$$
$$\doteq 1035\ N$$

Ex 4: An object is being towed by two ropes. The direction of forces of the ropes are $N20°W$ and $N30°E$. If the resultant force is 1000N due north, find the magnitude of the tensions of each rope.





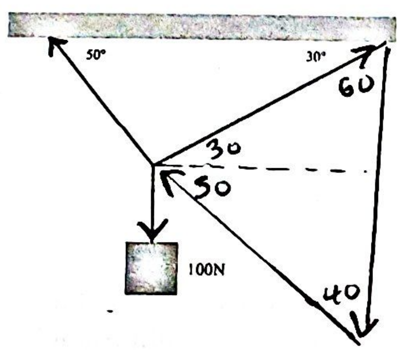
$$\frac{|T_1|}{\sin 20} = \frac{1000}{\sin 130°}$$

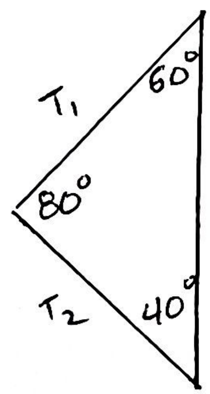
$$|T_1| \doteq 446.5N$$

$$\frac{|T_2|}{\sin 30°} = \frac{1000}{\sin 130}$$

$$|T_2| \doteq 652.N$$

Ex 5: A box with a force of 100N is hanging from two chains attached to an overhead beam at angles of 50° and 30° to the horizontal. Determine the tensions in the chains.



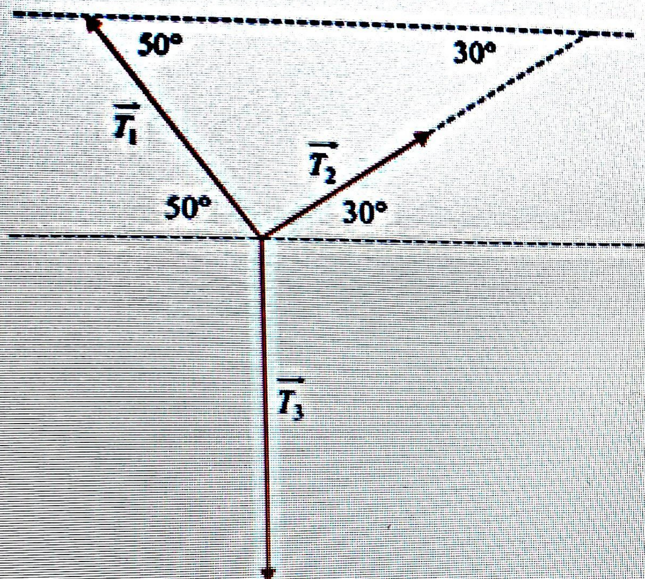


$$\frac{|T_1|}{\sin 40°} = \frac{100N}{\sin 80}$$

$$|T_1| \doteq 65.27 N$$

$$\frac{|T_2|}{\sin 60} = \frac{100}{\sin 80} \Rightarrow |T_2| \doteq 87.94 N$$

There is another way to do this. see the next page.



$$T_3 = 100N$$

$$R_x = T_2 \cos 30° - T_1 \cos 50° = 0 \Rightarrow T_2 = T_1(\cos 50°)/(\cos 30°)$$

$$R_y = T_1 \sin 50° + T_2 \sin 30° - 100 = 0 \Rightarrow$$

$$T_1 \sin 50° + T_1 \frac{\cos 50°}{\cos 30°} \sin 30° - 100 = 0$$

$$T_1 = \frac{100}{\sin 50° + \frac{\cos 50°}{\cos 30°} \sin 30°} \cong 87.94N$$

$$T_2 = \frac{T_1 \cos 50°}{\cos 30°} = \frac{87.94 \cos 50°}{\cos 30°} \cong 65.27N$$