2.1 – The Remainder Theorem (Day 1)

Recall: **Long Division**

34 → quotient
divisor ↙ 22)753 → dividend
748
5 → Remainder

$\frac{753}{22} = 34 + \frac{5}{22}$

OR

$753 = (34)(22) + 5$

Long Division of Polynomials: Long division can be used to divide a polynomial by a <u>binomial</u>

Example One - Divide the following polynomials

$$
\begin{array}{r}
x^2 + 3x - 19 \\
x+1\overline{)x^3 + 4x^2 - 16x - 14} \\
\underline{x^3 + x^2 \qquad\qquad} \\
3x^2 - 16x \qquad \\
\underline{3x^2 + 3x \qquad} \\
-19x - 14 \\
\underline{-19x - 19} \\
5
\end{array}
$$

$-14 - (-19)$
$= -14 + 19$
$= 15$

Step 1) Divide x^3 by x to get $\underline{x^2}$

Step 2) Multiply $x+1$ by $\underline{x^2}$ to get $\underline{x^3 + x}$. Write below.

Step 3) Subtract, bring down next term.

Step 4) Divide $3x^2$ by x to get $\underline{3x}$

Step 5) Multiply $x+1$ by $\underline{3x}$ to get $\underline{3x^2 + 3x}$ Write below.

Step 6) Subtract, bring down next term.

Step 7) Divide $-19x$ by x to get $\underline{-19}$

Step 8) Multiply $x+1$ by $\underline{-19}$ to get $\underline{-19x - 19}$ Write below.

Step 9) Subtract. Identify remainder.

A polynomial division statement with:

Dividend $P(x)$

Divisor $D(x)$ in the form of $x - b$ or $ax - b$

Quotient $Q(x)$

Remainder $R(x)$

Is written in *quotient form* as:

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

Ex:

$$\frac{x^3 + 4x^2 - 16x - 14}{x+1} = (x^2 + 3x - 19) + \frac{5}{x+1}$$

****The statement then can be used to check the division is $P(x) = D(x)Q(x) + R(x)$***

Example Two - Divide the following polynomials. Express the result in **quotient form.**

a) $f(x) = x^3 - 2x^2 + 4x - 1$ by $g(x) = x - 1$

$$
\begin{array}{r|l}
 & x^2 - x + 3 \\
\hline
x-1 & x^3 - 2x^2 + 4x - 1 \\
 & x^3 - x^2 \\
\hline
 & -x^2 + 4x \\
 & -x^2 + x \\
\hline
 & 3x - 1 \\
 & 3x - 3 \\
\hline
 & 2
\end{array}
$$

$$\therefore \frac{x^3 - 2x^2 + 4x - 1}{x - 1} = (x^2 - x + 3) + \frac{2}{x-1}$$

OR $f(x) = (x^2 - x + 3)(x - 1) + 2$

b) ** $j(x) = 3x^3 - 5x + 4$ by $k(x) = x + 3$

Please notice: x^2 term missing. Hence write $0x^2$

$$
\begin{array}{r|l}
 & 3x^2 - 9x + 22 \\
\hline
x+3 & 3x^3 + 0x^2 - 5x + 4 \\
 & 3x^3 + 9x^2 \\
\hline
 & -9x^2 - 5x \\
 & -9x^2 - 27x \\
\hline
 & 22x + 4 \\
 & 22x + 66 \\
\hline
 & -62
\end{array}
$$

$$\therefore \frac{3x^3 - 5x + 4}{x + 3} = (3x^2 - 5x + 4) - \frac{62}{x+3}$$

OR $j(x) = (3x^2 - 5x + 4)(x + 3) - 62$

Example Three- Find the values of $f(1)$ and $j(-3)$. What do you notice?

$f(1) = (1)^3 - 2(1)^2 + 4(1) - 1 = 2$

$j(-3) = 3(-3)^3 - 5(-3) + 4$

$= -62$

$f(1)$ is the remainder for dividing $f(x)$ by $x-1$

$j(-3)$ is the remainder for dividing $j(x)$ by $x+3$

Example Four- Without dividing, find the remainder:

a) $(x^2 + 2x + 4) \div (x - 2)$

$P(x) = x^2 + 2x + 4$

$P(2) = 2^2 + 2(2) + 4$

$= 4 + 4 + 4 = 12$

Remainder $= 12$

b) $(9x^3 - 6x^2 + 3x + 2) \div (3x - 1)$

$P(\frac{1}{3}) = 9(\frac{1}{3})^3 - 6(\frac{1}{3})^2 + 3(\frac{1}{3}) + 2$

$= \frac{8}{3}$

Example Five- If $P(x) = ax^3 + bx^2 - x + 3$ is divided by $x+1$, the remainder is 3. If $P(x)$ is divided by $(x+2)$, the remainder is -7. What are the values of a and b?

Given: $P(-1) = 3$ and $P(-2) = -7$

$P(-1) = 3 \Rightarrow a(-1)^3 + b(-1)^2 - (-1) + 3 = 3$

$-a + b + 1 + 3 = 3$

$\boxed{-a + b = -1}$

$P(-2) = -7 \Rightarrow a(-2)^3 + b(-2)^2 - (-2) + 3 = -7$

$-8a + 4b = -12 \Rightarrow -2a + b = -3$

$\left.\begin{array}{l} -a + b = -1 \\ -2a + b = -3 \end{array}\right\}$ USE SUBSTITUTION OR ELIMINATION

$-a - (-2a) = -1 - (-3)$

$\boxed{a = 2}$

sub $a = 2$ in $-a + b = -1$

$-2 + b = -1$

$\boxed{b = 1}$

$\therefore a = 2$ and $b = 1$

Remainder Theorem Practice

1. Without using long division, find each remainder:

(a) $(\underbrace{2x^2 + x - 6}_{P(x)}) \div (x + 2)$

$P(-2) = 2(-2)^2 + (-2) - 6$

$= 0$

$\therefore$ Rem $= 0$

(b) $(\underbrace{x^3 + 6x^2 - 4x + 2}_{P(x)}) \div (x + 1)$

Remainder $= (-1)^3 + 6(-1)^2 - 4(-1) + 2$

$= -1 + 6 + 4 + 2$

$= 11$

(c) $(x^3 + x^2 - 12x - 13) \div (x - 2)$

$R = 2^3 + 2^2 - 12(2) - 13$

$= -25$

(d) $(x^4 - x^3 - 3x^2 + 4x + 2) \div (x + 2)$

Rem $= (-2)^4 - (-2)^3 - 3(-2)^2 + 4(-2) + 2$

$= 16 + 8 - 12 - 8 + 2$

$= 6$

2. When $x^3 + kx^2 - 4x + 2$ is divided by $x + 2$ the remainder is 26, find k. $\Rightarrow$ $P(-2) = 26$ Solve for k

$(-2)^3 + k(-2)^2 - 4(-2) + 2 = 26$

$-8 + 4k + 8 + 2 = 26 \Rightarrow 4k = 24$

$\boxed{k = 6}$

3. When $2x^3 - 3x^2 + kx - 1$ is divided by $x - 1$ the remainder is 2, find k. $\Rightarrow$ $P(1) = 2$

$2(1)^3 - 3(1)^2 + k(1) - 1 = 2$

$2 - 3 + k - 1 = 2$

$\boxed{k = 4}$

ANSWERS: #1. (a) 0 (b) 11 (c) -25 (d) 6 #2. 6 #3. 4