# Arithmetic Sequences

A sequence where every successive term is found by ADDING THE SAME NUMBER is called **ARITHMETIC**.

**CHECK:** Pick any term, subtract the term before it. If the result is always the same no matter where in the sequence you begin, then the sequence is arithmetic.

**EXAMPLE 1**

9, 4, −1, −6, ...

ex: $4-9=-5$

$-6-(-1)=-5$

The terms are separated by a **COMMON DIFFERENCE** of −5

**EXAMPLE 2** – Find the general term of the following arithmetic sequence

2, 5, 8, 11, ... the common difference is 3 (let's call it "*d*")

↑ *a*

**Observe...**

| | | |
|---|---|---|
| 1st term | 2 | $a$ |
| 2nd term | 2 + 3 | $a + d$ |
| 3rd term | 2 + 3 + 3 | $a + 2d$ |
| 4th term | 2 +3 +3 +3 | $a + 3d$ |
| 5th term | 2 +3 +3 +3 +3 | $a + 4d$ |
| 6th term | 2 +3 +3 +3 +3 +3 | $a + 5d$ |

*Do you see the pattern?*

**CONCLUSION:** To find the general term of an arithmetic sequence

$$t_n = a + (n-1)d$$

where $a$ is the first term

$n$ is the number of the term

and $d$ is the common difference

# Arithmetic Sequences continued...

general term $t_n = a + (n-1)d$

**EXAMPLE 3** – Given the arithmetic sequence 8, 14, 20, 26, ...

a) Find the 20th term

$a = 8$

$d = 6$

general term:

$t_n = 8 + (n-1)(6)$

$t_n = 8 + 6n - 6$

$\boxed{t_n = 6n + 2}$

$t_{20} = 6(20) + 2$

the 20th term is 122

b) Which term is 236?

let $t_n = 236$

$\therefore 236 = 6n + 2$

$234 = 6n$

$n = 39$

the 39th term is 236.

**EXAMPLE 4** – The 3rd term of an arithmetic sequence is 8 while the 10th term of the same sequence is 4.5. Find the general term of the sequence.

$t_3 = 8$ $\quad t_{10} = 4.5$

but $\boxed{t_n = a + (n-1)d}$

$n=3$: $t_3 = a + 2d$ and $t_{10} = a + 9d$ ($n=10$)

$\boxed{a + 2d = 8}$ ① and $\boxed{a + 9d = 4.5}$ ②

② − ①: $7d = -3.5$

$d = -0.5$

sub in ②: $a + 9(-0.5) = 4.5$

$a - 4.5 = 4.5$

$a = 9$

$\therefore t_n = 9 + (n-1)(-0.5)$

$t_n = 9 - 0.5n + 0.5$

$t_n = -0.5n + 9.5$

# SEQUENCES

**SEQUENCE**
A set of numbers arranged in a specific order and following a specific pattern.

5, 7, 9, 11 ($t_1=5$, $t_2=7$)

2, 6, 18, 54 ($t_4=54$)

80, 40, 20, 10, ...

$t_n$ → Each "member" of the sequence is called a TERM. This is used to refer to any term in the sequence, also called the "general term." or nthterm

$n$ → Each "member" has a position in the sequence (1st, 2nd, 3rd, etc...). The letter $n$ stands for this position

$t_2$ is the 2nd term in a sequence

$t_{15}$ is the 15th term in a sequence

$t_{30}$ is the 30th term in a sequence

$t_n$ is ANY term in a sequence
the nth term / general term

$a$ → The FIRST TERM in a sequence has its own name! ( NOT $t_1$ )
↑ $t_1$ is okay too

## FUNCTIONS CAN BE SEQUENCES

**EXAMPLE 1** – Write the first 4 terms of the sequence:

a) $t_n = 2n+1$ (general term)
$t_1 = 2(1)+1$ $\therefore t_1 = 3$
$t_2 = 5$
$t_3 = 7$
$t_4 = 9$

b) $t_n = \frac{n}{n+1}$
$t_1 = \frac{1}{2}$ $t_3 = \frac{3}{4}$
$t_2 = \frac{2}{3}$ $t_4 = \frac{4}{5}$

**EXAMPLE 2** – Describe the pattern and predict what comes next

a) 3, 7, 11, 15, ...
add 4 to generate the next term
19, 23, 27
$t_n = 4 + t_{n-1}$

b) 2, 8, 32, 128, ...
multiply by 4 to generate the next term
512, 2048, 8192
$t_n = 4 \cdot t_{n-1}$

c) 1, 4, 9, 16, 25, ...
square each natural number
$t_n = n^2$
36, 49, 64

d) 1, 1, 2, 3, 5, 8, ...
add two consecutive terms to generate next term
13, 21, 34
$t_n = t_{n-1} + t_{n-2}$

recursive