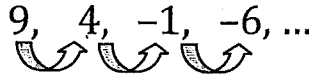


Arithmetic Sequences

A sequence where every successive term is found by **ADDING THE SAME NUMBER** is called **ARITHMETIC**.

CHECK: Pick any term, subtract the term before it. If the result is always the same no matter where in the sequence you begin, then the sequence is arithmetic.

EXAMPLE 1



ex: $4 - 9 = -5$
 $-6 - (-1) = -5$

The terms are separated by a **COMMON DIFFERENCE** of -5

EXAMPLE 2 - Find the **general term** of the following arithmetic sequence

2, 5, 8, 11, ... the common difference is 3 (let's call it "d")
 ↑
 a

Observe...

1 st term	2	a
2 nd term	2 + 3	a + d
3 rd term	2 + 3 + 3	a + 2d
4 th term	2 + 3 + 3 + 3	a + 3d
5 th term	2 + 3 + 3 + 3 + 3	a + 4d
6 th term	2 + 3 + 3 + 3 + 3 + 3	a + 5d

Do you see the pattern?

CONCLUSION: To find the general term of an arithmetic sequence

$$t_n = a + (n-1)d$$

where **a** is the first term

n is the number of the term

and **d** is the common difference

Arithmetic Sequences continued...

general term
 $t_n = a + (n-1)d$

EXAMPLE 3 - Given the arithmetic sequence 8, 14, 20, 26, ...

a) Find the 20th term

$$a = 8$$

$$d = 6$$

general term:

$$t_n = 8 + (n-1)(6)$$

$$t_n = 8 + 6n - 6$$

$$\boxed{t_n = 6n + 2}$$

$$t_{20} = 6(20) + 2$$

the 20th term is 122

b) Which term is 236?

$$\text{let } t_n = 236$$

$$\therefore 236 = 6n + 2$$

$$234 = 6n$$

$$n = 39$$

the 39th term is 236.

EXAMPLE 4 - The 3rd term of an arithmetic sequence is 8 while the 10th term of the same sequence is 4.5. Find the general term of the sequence.

$$t_3 = 8$$

$$t_{10} = 4.5$$

but $\boxed{t_n = a + (n-1)d}$

$n=3$

$$t_3 = a + 2d$$

$$\boxed{a + 2d = 8} \text{ (1)}$$

and

$$t_{10} = a + 9d$$

$$\boxed{a + 9d = 4.5} \text{ (2)}$$

$n=10$

$$\text{(2) - (1): } 7d = -3.5$$

$$d = -0.5$$

$$\text{sub in (2): } a + 9(-0.5) = 4.5$$

$$a - 4.5 = 4.5$$

$$a = 9$$

$$\therefore t_n = 9 + (n-1)(-0.5)$$

$$t_n = 9 - 0.5n + 0.5$$

$$t_n = -0.5n + 9.5$$

SEQUENCES

SEQUENCE

A set of numbers arranged in a specific order and following a specific pattern.

$$5, 7, 9, 11$$

$t_1 = 5$ $t_2 = 7$

$$2, 6, 18, 54$$

$t_4 = 54$

$$80, 40, 20, 10, \dots$$

$t_n \rightarrow$ Each "member" of the sequence is called a TERM. This is used to refer to any term in the sequence, also called the "general term." or n^{th} term

$n \rightarrow$ Each "member" has a position in the sequence (1st, 2nd, 3rd, etc...). The letter n stands for this position

t_2 is the 2nd term in a sequence

t_{15} is the 15th term in a sequence

t_{30} is the 30th term in a sequence

t_n is ANY term in a sequence
the n^{th} term / general term

$a \rightarrow$ The FIRST TERM in a sequence has its own name! (NOT t_1)

t_1 is okay too

FUNCTIONS CAN BE SEQUENCES

EXAMPLE 1 - Write the first 4 terms of the sequence:

a) $t_n = 2n + 1$ (general term)

$$t_1 = 2(1) + 1 \quad \therefore t_1 = 3$$

$$t_2 = 5$$

$$t_3 = 7$$

$$t_4 = 9$$

b) $t_n = \frac{n}{n+1}$

$$t_1 = \frac{1}{2}$$

$$t_2 = \frac{2}{3}$$

$$t_3 = \frac{3}{4}$$

$$t_4 = \frac{4}{5}$$

EXAMPLE 2 - Describe the pattern and predict what comes next

a) 3, 7, 11, 15, ...
add 4 to generate the next term

$$19, 23, 27$$

$$t_n = 4 + t_{n-1}$$

b) 2, 8, 32, 128, ...
multiply by 4 to generate the next term

$$512, 2048, 8192$$

$$t_n = 4 \cdot t_{n-1}$$

c) 1, 4, 9, 16, 25, ...
square each natural number
 $t_n = n^2$
36, 49, 64

d) 1, 1, 2, 3, 5, 8, ...
add two consecutive terms to generate next term

$$13, 21, 34$$

recursive $t_n = t_{n-1} + t_{n-2}$

