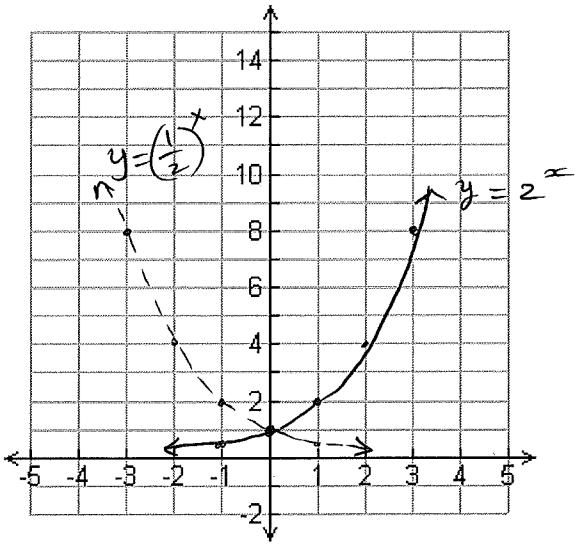
Day 1: Review of Prerequisite Skills

Exponential Functions

Graph $y = 2^x$ and $y = (\frac{1}{2})^x$



Why does it not have any x-intercepts?

$y=0$ is the horizontal asymptote.

Would $y = 2^x - 4$ have x-intercept? Explain.

Yes. $y=-4$ is the HA.
The graph is increasing [and NOT reflected]

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y > 0\}$ $y=2^x, y=(\frac{1}{2})^x$

$\{y \in \mathbb{R} \mid y > -4\}$

Exponent laws

- **Product rule** $a^x a^y = a^{x+y}$
- **Quotient rule** $\frac{a^x}{a^y} = a^{x-y}$
- **Power rule** $(a^x)^y = a^{xy}$
- **Zero exponent**

$a^0 = 1$, $-a^0 = -1$, $(-a)^0 = 1$

- **Negative exponent**

$a^{-x} = \frac{1}{a^x}$, $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$, $\frac{1}{a^{-x}} = a^x$

Inverse functions:

$f(x) = \{(1,3), ((-1,5)), (2,4), (5,7)\}$

Domain: $\{-1, 1, 2, 5\}$

Range: $\{3, 4, 5, 7\}$

$f^{-1}(x) = \{(3,1), (5,-1), (4,2), (7,5)\}$

Domain: $\{3, 4, 5, 7\}$

Range: $\{-1, 1, 2, 5\}$

Transformations

In general:

$$y = af(k(x - d)) + c$$

- $a < 0$: reflection in x-axis.
- $|a| > 1$: vertically stretched by a factor of $|a|$
- $0 < |a| < 1$: vertically compressed by a factor of $|a|$

$k < 0$: reflection in y-axis

$|k| > 1$: Horizontally comp. by a factor of $\frac{1}{|k|}$

$0 < |k| < 1$: Horizontally comp by a factor of $\frac{1}{|k|}$

Horizontally translated 'd' units to the right ($d > 0$) left ($d < 0$)

Vertically translated c units up ($c > 0$) down ($c < 0$)

Mapping

$$(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

Quadratic Equations

- Solve by factoring: $x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6, \ 4$

$x = \{-6, 4\}$

- Solve by quadratic formula: $y^2 + 6y - 5 = 0$ $\quad a=1 \quad b=6 \quad c=-5$

$$y = \frac{-6 \pm \sqrt{6^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{56}}{2}$$

$$= \frac{-6 \pm \sqrt{4}\sqrt{14}}{2} = \frac{-6 \pm 2\sqrt{14}}{2} = -3 \pm \sqrt{14}$$

Simplify radical expression

a. $\sqrt{8} = \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

b. $\sqrt{32} = \sqrt{16}\sqrt{2}$

$= 4\sqrt{2}$

c. $\sqrt{128} = \sqrt{64}\sqrt{2}$

$= 8\sqrt{2}$

d. $\frac{2 \pm \sqrt{24}}{2}$

$= \frac{2 \pm \sqrt{4}\sqrt{6}}{2} = \frac{2 \pm 2\sqrt{6}}{2}$

$= 1 \pm \sqrt{6}$