Day 10: 1.6 – Slopes of Tangents & Instantaneous Rate of Change

Example One: Let's Investigate!

A golf ball lying on the grass is hit so that its initial vertical velocity is 25m/s. The height, h, in metres, of the ball after t seconds can be modelled by the quadratic function $h(t) = -4.9t^2 + 25t$.

a) Copy and complete the table for $h(t)$.

Interval	Δh	Δt	Average rate of change: $\frac{\Delta h}{\Delta t}$
$1 \le t \le 2$	$= h(2) - h(1)$ $= [-4.9(2)^2 + 25(2)] - [-4.9(1)^2 + 25(1)]$ $= 30.4 - 20.1$ $= 10.3$	$= 2 - 1$ $= 1$	$= \frac{10.3}{1}$ $= 10.3 m/s$
$1 \le t \le 1.1$	$= h(1.1) - h(1)$ $= [-4.9(1.1)^2 + 25(1.1)] - 20.1$ $= 21.571 - 20.1$ $= 1.471$	$= 1.1 - 1$ $= 0.1$	$= \frac{1.471}{0.1}$ $= 14.71$ m/s
$1 \le t \le 1.01$	$= [-4.9(1.01)^2 + 25(1.01)] - 20.1$ $= 20.25 - 20.10$ $= 0.15$	$= 1.01 - 1$ $= 0.01$	$= \frac{0.15}{0.01}$ $= 15$ m/s
$1 \le t \le 1.001$	$= [-4.9(1.001)^2 + 25(1.001)] - 20.1$ $= 20.115 - 20.10$ $= 0.015$	$= 1.001 - 1$ $= 0.001$	$= \frac{0.015}{0.001}$ $= 15$ m/s

b) Explain how the time intervals in the first column are changing

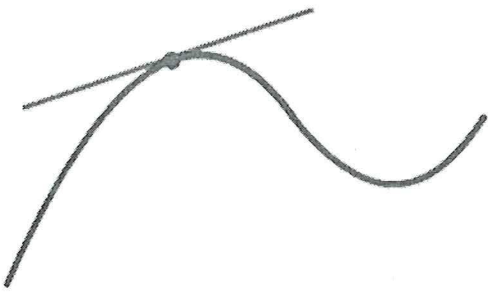
Decreasing each time [getting smaller and smaller]

c) Describe how the average rate of change values in the fourth column are changing in relation to the time intervals

AROC are getting closer and closer to 15

Recall: rate of change is a measure of how quickly one quantity (the dependent variable) changes with respect to another quantity (the independent variable)

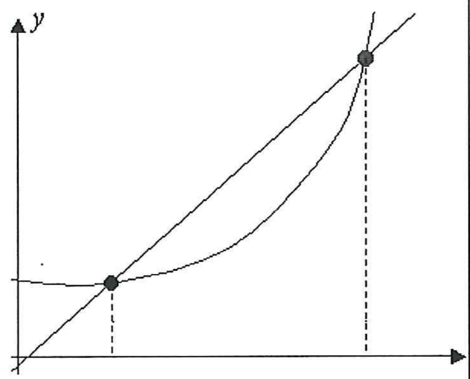
Instantaneous Rate of Change represents the rate of change at a specific point.



Tangent is a line that touches a curve at a single point.

The instantaneous rate of change corresponds to the **slope of a tangent** to a point on the curve.

Relationship between slope of secants and slope of a tangent:



- As point Q becomes very close to tangent point P, the slope of the secant line becomes closer to *(or approaches, denoted as →)*

As Q → P:

- The slope of secant PQ → is slope of the tangent.
- The average rate of change between P and Q → instantaneous rate of change.

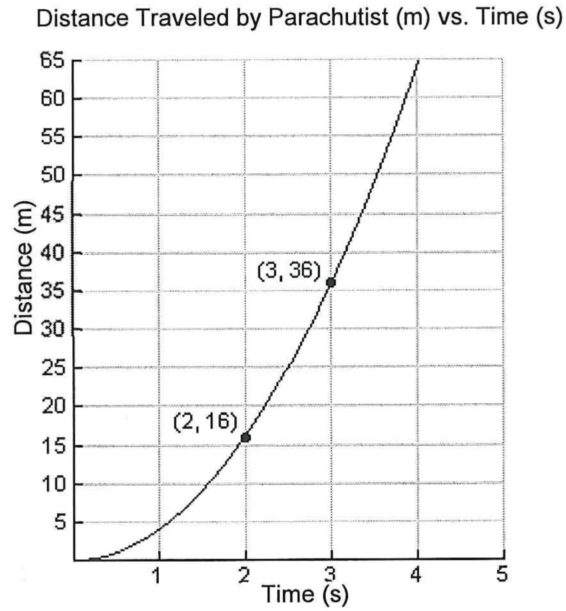
An <u>approximated</u> value for instantaneous rate of change at a point can be determined using:

- **<u>A Graph:</u>** By sketching the tangent to a point P and then calculating the slope between point P and another point on the tangent line
- **<u>A Table of Values:</u>** By estimating the slope between a point P and a nearby point in the table
- **<u>An Equation:</u>** By estimating the slope using a very short interval between a point P and a nearby second point using the equation

Example Two: **Using a graph**

The graph shows the distance traveled by a parachutist in the first 5 seconds after jumping out of a helicopter.

Estimate the parachutist's velocity 2 seconds after jumping by sketching a tangent. *(Note: velocity is speed with direction)* (2, 16) (2.3, 20)



$$IROC = \frac{20-16}{2.3-2}$$

$$= \frac{4}{0.3}$$

$$\doteq 13.3 \text{ m/s.}$$

Example Three: **Using a table of values**

A data recorder measured the distance travelled by the parachutist every half second for the first three seconds of the jump.

Estimate the parachutist's velocity 2 seconds after jumping by using the table of values

Time (s)	Distance (m)
0.0	0
0.5	1
1.0	4
1.5	9
2.0	16
2.5	25
3.0	36

(Preeciding interval) $1.5 \le t \le 2.00$

$$m_1 = \frac{16-9}{2-1.5} = \frac{7}{0.5} = 14$$

(following interval) $2 \le t \le 2.5$

$$m_2 = \frac{25-16}{2.5-2} = \frac{9}{0.5} = 18$$

$$\therefore m = \frac{m_1 + m_2}{2} = \frac{14+18}{2} = 16 \text{ m/s.}$$

Example Four: **Using an equation**

The distance travelled by the parachutist, $d(t)$, after t seconds can be modelled by the function $d(t) = 4t^2$

Estimate the parachutist's velocity 2 seconds after jumping by calculating the speed over a very small interval near 2 seconds.

Interval	Δt	Δd	AROC
$2 \leq t \leq 2.1$	$= 2.1 - 2$ $= 0.1$	$= [4(2.1)^2] - [4(2)^2]$ $= 17.64 - 16$ $= 1.64$	$= \frac{1.64}{0.1}$ $= 16.4$ m/s
$2 \leq t \leq 2.01$	$= 2.01 - 2$ $= 0.01$	$= [4(2.01)^2] - [4(2)^2]$ $= 16.1604 - 16$ $= 0.1604$	$= \frac{0.1604}{0.01}$ $= 16.04$
$2 \leq t \leq 2.001$	$= 2.001 - 2$ $= 0.001$	$= [4(2.001)^2] - [4(2)^2]$ $= 16.016004 - 16$ $= 0.016004$	$= \frac{0.016004}{0.001}$ $= 16.004$

Let's Reflect:

Which of the three methods do you think would be the most reliable?

Using equation as Δt can be very small and we can get accurate y coordinates compared to estimating.

$$IROC = \frac{f(a+h) - f(a)}{h} \quad [h \to 0]$$