SOLVING TRIANGLES

To "**solve**" a triangle means to find all sides and all angles
Unless otherwise specified, round angles to nearest degree and round lengths/ratios to one decimal place.

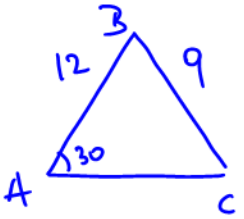
Hints for Solving Trig Word Problems

1. Draw and label a diagram
2. Choose the rule or law needed
3. Solve for the unknown
4. Write a concluding sentence including units.

Type of Triangle	Information Given	Rule/Law
Right Triangle	Any 2 pieces of info (except 2 angles only)	SOH CAH TOA Pythagorean theorem
Oblique (i.e. triangle which contains no right angle, and which may or may not contain an obtuse angle)	AAS, ASA SSA SSS, SAS AAA	Sine Law Sine Law, ambiguous? Cosine Law ** Can't solve without at least one side

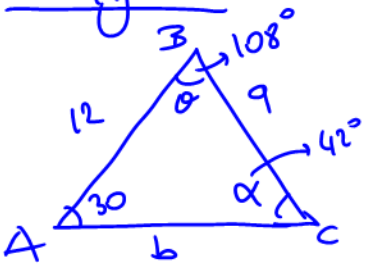
REVIEW OF SINE LAW – AMBIGUOUS CASE

Ex1. In ΔABC, ∠A = 30º, c = 12cm and a = 9cm. Determine the number of triangles possible. Solve the triangle(s) if possible.



if $12\sin 30 < BC < 12$, then there'll be 2 triangles.

Triangle 1



$\frac{\sin\alpha}{12} = \frac{\sin 30}{9}$

$\sin\alpha = \frac{12}{9}\sin 30$

$\sin^{-1}\left(\frac{12}{9}\sin 30\right) = 9$

$\alpha = 42°$

Solve for $\angle B°$ and b,

$180 - (30 + 42) = \theta$

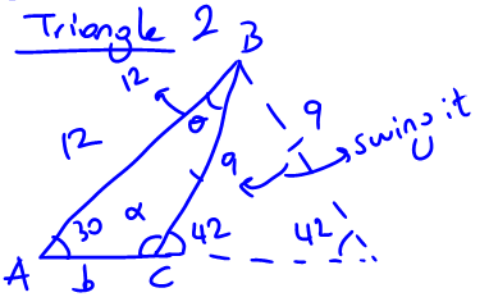
$108° = \theta$

$\frac{b}{\sin 108} = \frac{9}{\sin 30}$

$b = \frac{9\sin 108}{\sin 30}$

$b = 17.1\text{cm}$

Triangle 2



$\alpha + 42 = 180$

$\alpha = 138°$

$\theta = 180 - (30 + 138)$

$\theta = 12°$

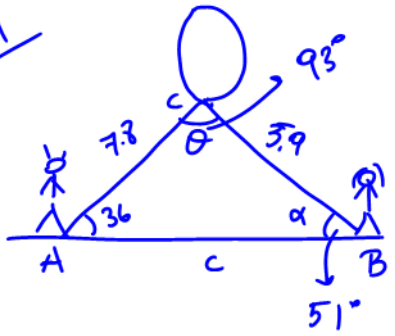
$\frac{b}{\sin 12} = \frac{9}{\sin 30}$

$b = \frac{9\sin 12}{\sin 30}$

$b = 3.7\text{cm}$

Ex2. Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of a cloud. Albert's rope is 7.8m long and makes an angle of 36° with the ground. Belle's rope is 5.9m long. How far, to the nearest tenth of a metre, is Albert from Belle?

1



$\frac{\sin\alpha}{7.8} = \frac{\sin 36}{5.9}$

$\sin\alpha = \frac{7.8\sin 36}{5.9}$

$\sin^{-1}\left(\frac{7.8}{5.9}\sin 36\right) = \alpha$

$\alpha = 51°$

$\theta = 180 - (36 + 51)$

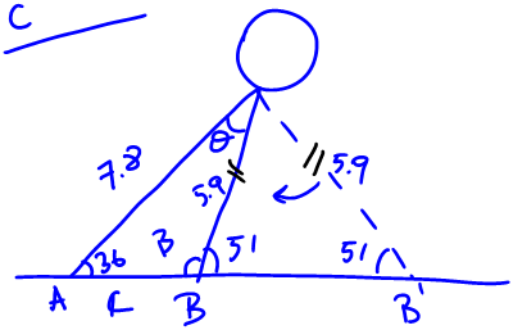
$\theta = 93°$

$\frac{c}{\sin 93} = \frac{5.9}{\sin 36}$

$c = \frac{5.9\sin 93}{\sin 36}$

$c \doteq 10m$

2

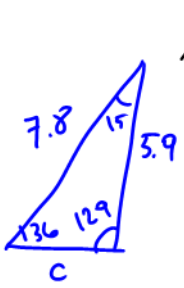


$\beta = 180 - 51$

$\beta = 129°$

$\theta = 180 - (36 + 129)$

$\theta = 15°$



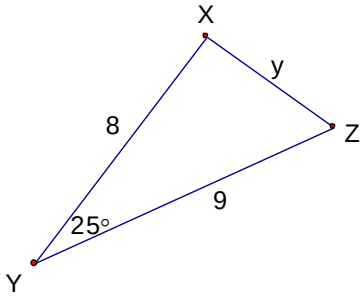
$\frac{c}{\sin 15} = \frac{5.9}{\sin 36}$

$c = \frac{5.9\sin 15}{\sin 36}$

$c \doteq 2.6m$

WARM UP

Ex1. Find the value of side y.



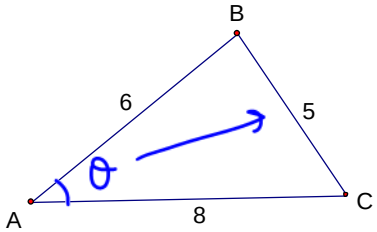
$y^2 = 8^2 + 9^2 - 2.8.9\cos 25$

$y^2 = 64 + 81 - 130.5$

$y^2 = 14.5$

$y = 3.8$

Ex2. Find the value of <A.



$5^2 = 6^2 + 8^2 - 2.6.8.\cos\theta$

$25 = 36 + 64 - 96\cos\theta$

$-75 = -96\cos\theta$

$\frac{75}{96} = \cos\theta$

$\theta = \cos^{-1}\left(\frac{75}{96}\right) \Rightarrow \theta = 51°$

Ex3. Mitchell wants his 8m wide house to be heated with a solar hot – water system. The tubes form an array that is 5.1 m long. In order for the system to be effective, the array must be installed on the south side of the roof and the roof needs to be inclined by 60°. If the north side of the roof is inclined more than 40°, the roof will be too steep for Mitchell to install the system himself. Will Mitchell be able to install this system by himself?

Solve for "b"

$b^2 = (5.1)^2 + (8)^2 - 2(5.1)(8)\cos 60$

$b^2 = 26.01 + 64 - 40.8$

$b^2 = 49.21$

$b \doteq 7\text{m}$

Solve for θ

$\frac{\sin\theta}{5.1} = \frac{\sin 60}{7}$

$\sin\theta = \frac{5.1\sin 60}{7}$

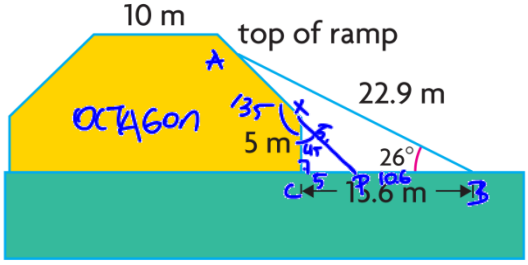
$\sin^{-1}\left(\frac{5.1\sin 60}{7}\right) = \theta$

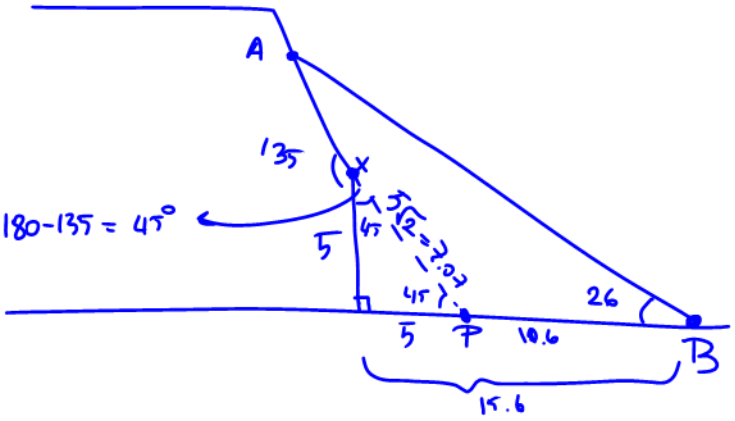
$\theta = 39°$

$\therefore$ Since it's inclined about 39°, he'll be able to do it himself.

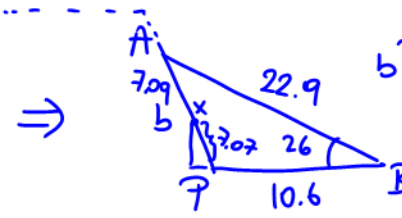
CHALLENGE

Ex4. Determine the distance from the top of the ramp to the roof.





each interior angle is 135°



$\Rightarrow b^2 = (22.9)^2 + (10.6)^2 - 2(22.9)$

$b^2 = 200.4$

$b = 14.16$

$\overline{AX} = 14.16 - 7.07$

$\overline{AX} = 7.09\text{ m}$

$d = 10 - \overline{AX}$

$= 10 - 7.09$

$= 2.9$

$\overline{PX} = 5\sqrt{2} = 7.07$

$\therefore$ The top of the ramp is about 2.9m.