1. There are 2 rational expressions, P/Q and R/S, where $Q = x^2 - 9$, $R = x + 1$, and $S = x^2 + x - 6$. If $P/Q \div R/S = A/B$, where $A = 4x^2 - 13x + 10$, determine an expression for P and B.

$$\frac{P}{Q} \div \frac{R}{S} = \frac{A}{B}$$

$$\frac{P}{x^2-9} \div \frac{(x+1)}{x^2+x-6} = \frac{4x^2-13x+10}{B}$$

$$\frac{P}{(x-3)(x+3)} \div \frac{(x+1)}{(x-2)(x+3)} = \frac{(x-2)(4x-5)}{B}$$

$$\frac{P}{(x-3)(x+3)} \cdot \frac{(x-2)(x+3)}{(x+1)} = \frac{(x-2)(4x-5)}{B}$$

$\therefore P = 4x-5$

$B = (x-3)(x+1)$

$4x^2-13x+10$

$=4x^2-8x-5x+10$

$=4x(x-2)-5(x-2)$

$=(x-2)(4x-5)$

M	A	N
40	-13	-5, -8

2. Rowing at 8 km/h in still water, Rina and Bhanu take 16 hours to row 39 km down a river and 39 km back. Find the speed of the current.

Let "c" be the speed of the current.

	distance	speed	time
down (faster)	39	8 + c	$\frac{39}{8+c}$
up (slower)	39	8 − c	$\frac{39}{8-c}$

Total time = Time$_{down}$ + Time$_{up}$

$$16 = \frac{(8-c)\,39}{(8-c)(8+c)} + \frac{(8+c)\,39}{(8+c)(8-c)} \qquad \text{LCD } (8+c)(8-c)$$

$$16 = \frac{39(8-c)}{(8-c)(8+c)} + \frac{39(8+c)}{(8-c)(8+c)}$$

$$16 = \frac{312 - 39c + 312 + 39c}{64 - c^2}$$

$$16^{(64-c^2)} = \frac{624}{64-c^2} \cdot (64-c^2)$$

$$1024 - 16c^2 = 624$$

$$1024 - 624 = 16c^2$$

$$\frac{400}{16} = \frac{16c^2}{16}$$

$$\sqrt{25} = \sqrt{c^2}$$

$$c = \mp 5$$

$\therefore$ The speed of the current is 5 km/h

3. A rectangular prism has $length = \frac{2x-5}{x+4}, width = \frac{3x+2}{3x-1}$ *and* $height = \frac{x+4}{3x+1}$ all in metres.

a) Determine a simplified expression for the volume of the rectangular prism. Express your answer as a quotient of two polynomials in standard ~~(not factored)~~ form, and state any restrictions.

b) Determine the volume when x = 4 metres.

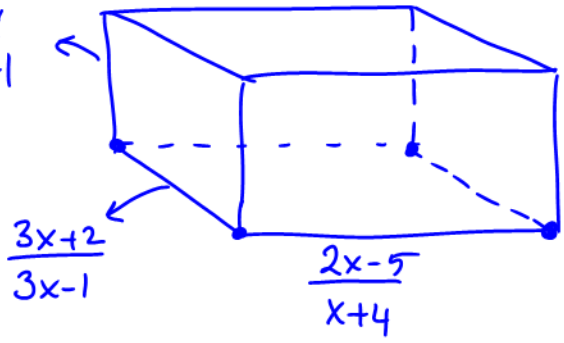
a) $V = \frac{(2x-5)}{(x+4)} \cdot \frac{(3x+2)}{(3x-1)} \cdot \frac{(x+4)}{(3x+1)}$

$x \neq -4, -\frac{1}{3}, \frac{1}{3}$

$= \frac{(2x-5)(3x+2)}{(3x-1)(3x+1)}$

$= \frac{6x^2+4x-15x-10}{9x^2-1}$

$= \frac{6x^2-11x-10}{9x^2-1}$



b) $V(4) = \frac{6(4)^2-11(4)-10}{9(4)^2-1}$

$= \frac{6 \cdot 16 - 44 - 10}{9 \cdot 16 - 1}$

$= \frac{96-54}{144-1}$

$= \frac{42}{143}$

4. There are 2 rational expressions, P/Q and R/S, where $Q = x^2 - 9$, $R = x + 1$, and $S = x^2 + x - 6$. If P/Q + R/S = A/B, where $A = 4x^2 - 12x + 5$, determine an expression for P and B.

$\frac{P}{Q} + \frac{R}{S} = \frac{A}{B}$

$\frac{P}{x^2-9} + \frac{(x+1)}{x^2+x-6} = \frac{4x^2-12x+5}{B}$

$\frac{P}{(x-3)(x+3)} + \frac{(x+1)}{(x-2)(x+3)} = \frac{(2x-1)(2x-5)}{B}$

FOIL

$\frac{P(x-2)}{(x-3)(x+3)(x-2)} + \frac{(x+1)(x-3)}{(x-2)(x+3)(x-3)} = \frac{(2x-1)(2x-5)}{B}$

this is not getting us anywhere. convert it back to original form.

equal

$\frac{P(x-2) + x^2-2x-3}{(x-3)(x+3)(x-2)} = \frac{(2x-1)(2x-5)}{B}$

$P(x-2) + x^2 - 2x - 3 = 4x^2 - 12x + 5$

$P(x-2) = 4x^2 - 12x + 5 - x^2 + 2x + 3$

$\frac{P(x-2)}{(x-2)} = \frac{3x^2-10x+8}{(x-2)}$ → let's factor it to see if we can simplify

$\therefore P = \frac{(3x-4)(x-2)}{(x-2)}$

$P = (3x-4)$

Rough Work

$4x^2-12x+5$ M: 20 | A: -12 | N: -2, -10

$= \frac{(4x-2)(4x-10)}{4}$

$= \frac{2(2x-1)(2)(2x-5)}{5}$

$= (2x-1)(2x-5)$

LCD

$(x-3)(x+3)(x-2)$

$(x-2)(x+3)(x-3)$

$3x^2-10x+8$ M: 24 | A: -10 | N: -6, -4

$= \frac{(3x-4)(3x-6)}{3}$

$= \frac{(3x-4)(3)(x-2)}{3}$

$= (3x-4)(x-2)$

5. On the 42 km go-kart course, Arshia drives 0.4 km/h faster than Sarah, but has engine trouble and stops for ½ hour. She arrives 15 minutes after Sarah at the end of the course. How fast did each girl drive?

Prep

	distance	speed	time
Sarah	42	s	$\frac{42}{s}$
Arshia	42	$s+0.4$	$\frac{42}{s+0.4} + \frac{1}{2}$ → had an engine trouble

let "s" be the speed of Speedy Sarah

More Prep

We know that Arshia took another 15 min which is $15/60$ hour; therefore, we can say → 1/4

Arshia's time equals Sarah's time + $\frac{15}{60}$

$$\frac{42}{s+0.4} + \frac{1}{2} = \frac{42}{s} + \frac{1}{4}$$

$$\frac{1}{2} - \frac{1}{4} = \frac{42}{s} - \frac{42}{s+0.4}$$

$$\frac{1}{4} = \frac{42(s+0.4)}{s(s+0.4)} - \frac{42s}{s(s+0.4)}$$

$$\frac{1}{4} = \frac{42s + 16.8 - 42s}{s^2+0.4s}$$ cross mult.

$s^2+0.4s = 67.2$

$s^2+0.4s-67.2=0$ → use quadratic formula

$s = 8$ or $s = -8.4$

∴ Sarah 8 km/h

Arshia 8.4 km/h

PRACTICE

1. An open cardboard box with a square base with a side of x cm has a volume of 100 cm^3.
   a. Express the height of the box, h, in terms of x.
   b. Express the surface area of the 5 sides of the box in terms of x.

2. A rectangular board has an area of 6000 cm^2 and a width of *w* cm.
   a) Write an expression for the length of the board.
   b) Write an expression for the perimeter of the board.
   c) If the width is increased by x cm, write an expression for the new perimeter of the board.
   d) Write an expression for the change in perimeter ($P_2 - P_1$).

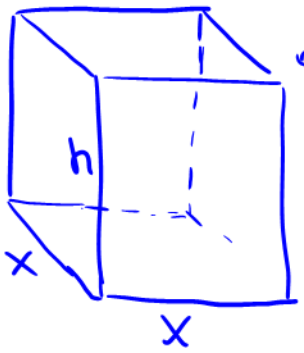
3. One lap of a motorcycle race is 650 m. At the start of the race, Genna sets off 4 seconds after Tom does, but she drives her motorcycle 5 m/s faster and finishes the lap 2.5 seconds sooner than he does. Find their speeds.

4. Marissa and Jovanna enter a 200-km bike race. Marissa cycles 5 km/h faster than Jovanna, but her bicycle gets a flat tire, which takes ½ hour to repair. If the 2 girls finish the race in a tie, how fast was each girl cycling?

7. An open cardboard box with a square base with a side of x cm has a volume of 100 cm3.

a) Express the height of the box, h, in terms of x.

b) Express the surface area of the 5 sides of the box in terms of x.

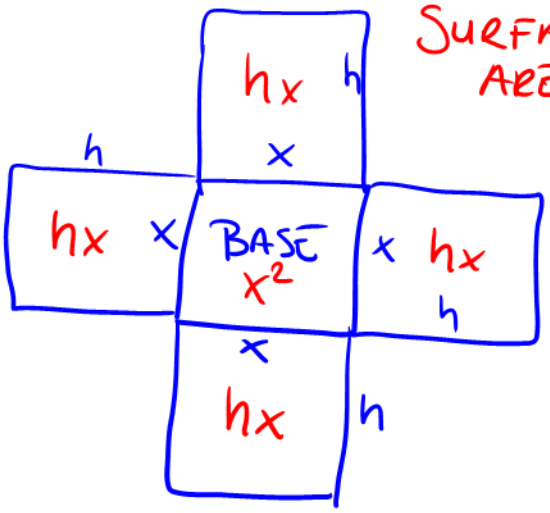


a) Volume = Area$_{base}$ x height

$100 = x^2 h$

$\frac{100}{x^2} = h$

b)



SURFACE AREA $= 4hx + x^2$ sub $\frac{100}{x^2}$ for "h"

$= \frac{400x}{x^2} + x^2$ $x \neq 0$

$= \frac{400}{x} + \frac{x^2(x)}{1(x)}$

$= \frac{400 + x^3}{x}$ $\therefore \frac{x^3 + 400}{x}$

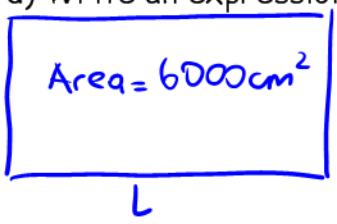
2. A rectangular board has an area of 6 000 cm2 and a width of w cm.

a) Write an expression for the length of the board.

b) Write an expression for the perimeter of the board.

c) If the width is increased by x cm, write an expression for the new perimeter of the board.

d) Write an expression for the change in perimeter (P2 – P1).



a) Area = Length x Width

$6000 = L \times w$

$\therefore L = \frac{6000}{w}$

b) Perimeter $= 2(L+w)$

$P_1 = 2\left(\frac{6000}{w} + w\right)$

$= 2\left(\frac{6000+w^2}{w}\right)$

$\therefore P_1 = \frac{2w^2+12000}{w}$

c) $P_2 = \frac{2(w+x)^2+12000}{w+x} \Rightarrow P_2 = \frac{2w^2+4wx+2x^2+12000}{w+x}$

d) $P_2 - P_1 = \frac{w(2w^2+4wx+2x^2+12000)}{(w)(w+x)} - \frac{(2w^2+12000)(w+x)}{w(w+x)}$

$LCD = w(w+x)$

$= \frac{2w^3 + 4w^2x + 2x^2w + 12000w - 2w^3 - 2w^2x - 12000w - 12000x}{w(w+x)}$

$= \frac{2w^2x + 2x^2w - 12000x}{w(w+x)}$

3. One lap of a motorcycle race is 650 m. At the start of the race, Genna sets off 4 seconds after Tom does, but she drives her motorcycle 5 m/s faster and finishes the lap 2.5 seconds sooner than he does. Find their speeds.

	Distance	Speed	Time
Tom	650	V	$\frac{650}{V}$
Genna	650	$V+5$	$\frac{650}{V+5}$ (+4)

Let "V" be the speed of Tom.

$d = Vt$

(+4) → couldn't race for 4 seconds

Genna's time is 2.5 seconds less than Tom

Genna's time = Tom's time − 2.5

$$\frac{650}{V+5} + 4 = \frac{650}{V} - 2.5$$ collect terms on LHS

$$\frac{650}{V+5} - \frac{650}{V} + 6.5 = 0 \qquad LCD = V(V+5)$$

$$\frac{650(V)}{(V+5)(V)} - \frac{650(V+5)}{V(V+5)} + 6.5\frac{V(V+5)}{V(V+5)} = 0$$

$$\frac{650V - 650V - 3250 + 6.5V^2 + 32.5V}{V(V+5)} = 0$$ ← multiply each side by $V(V+5)$ to get rid of it.

$$6.5V^2 + 32.5V - 3250 = 0$$ reduce by 5

$$1.3V^2 + 6.5V - 650 = 0$$

Use quadratic formula $a = 1.3 \quad b = 6.5 \quad c = -650$

$$V_{1,2} = \frac{-6.5 \mp \sqrt{(6.5)^2 - 4(1.3)(-650)}}{2(1.3)}$$

$$V_{1,2} = \frac{-6.5 \mp \sqrt{3422.25}}{2.6}$$

$$V_1 = \frac{-6.5 + 58.5}{2.6} = \frac{52}{2.6} = 20 \text{ m/s}$$

$$V_2 = \frac{-6.5 - 58.5}{2.6} = -25 \text{ m/s}$$ can't be negative

∴ Tom's speed is 20 m/s
Genna's speed is 25 m/s

4. Marissa and Jovanna enter a 200-km bike race. Marissa cycles 5 km/h faster than Jovanna, but her bicycle gets a flat tire, which takes $\frac{1}{2}$ hour to repair. If the 2 girls finish the race in a tie, how fast was each girl cycling?

	Distance	Speed	Time → raced
J	200	V	$\frac{200}{V}$
M	200	$V+5$	$\frac{200}{V+5} + 0.5$ → late start

let "V" be the speed of Jovanna since Marissa is 5 km/h faster whatever her speed is.

$$\frac{200}{V} = \frac{200}{V+5} + \frac{0.5(V+5)}{1(V+5)}$$

$$\frac{200}{V} \times \frac{200+0.5V+2.5}{V+5}$$ Cross multiply

$$200V + 1000 = 200V + 0.5V^2 + 2.5V$$

$$-0.5V^2 - 2.5V + 1000 = 0$$ Use quadratic formula to solve for "V"

$$V_{1,2} = \frac{-(-2.5) \mp \sqrt{(-2.5)^2 - 4(-0.5)(1000)}}{2(-0.5)}$$ $a = -0.5$ $b = -2.5$ $c = 1000$

$$V_{1,2} = \frac{2.5 \mp \sqrt{2006.25}}{-1}$$

$$V_1 = \frac{2.5 + 44.79}{-1} = -46.79$$

$$V_2 = \frac{2.5 - 44.79}{-1} = 42.29$$

∴ The speed of Jovanna is 42.3 km/h
Marissa is 47.3 km/h